

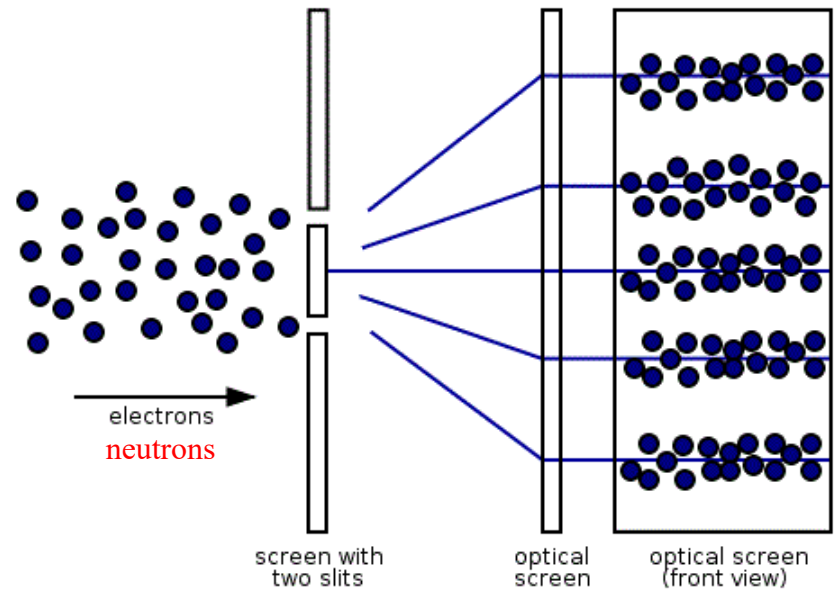
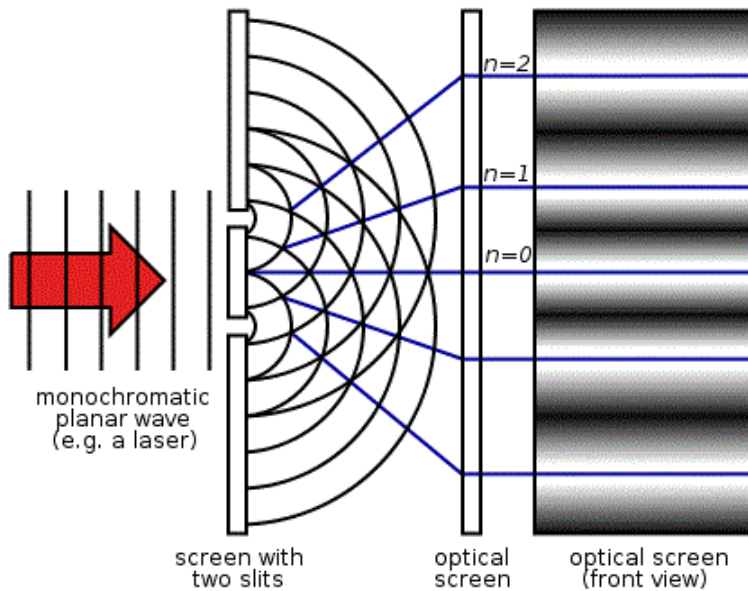
**Fundamental Phenomena in Quantum Mechanics
studied with Matter-Wave Optical Setup
--- Quantum Cheshire-Cat and Uncertainty Relations ---**

Yuji HASEGAWA

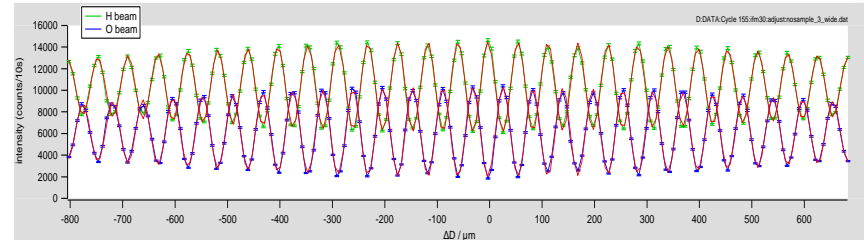
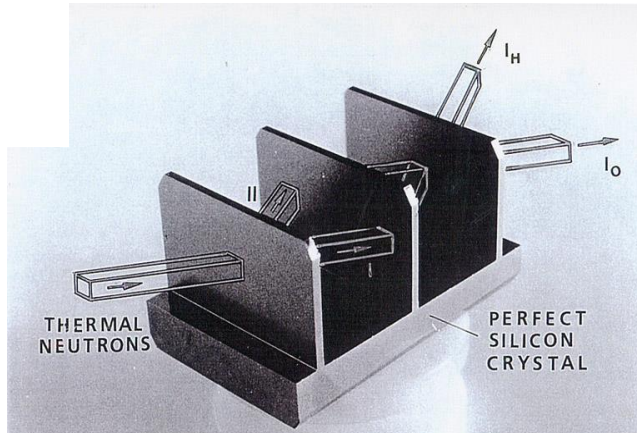
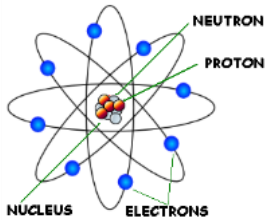
Atominsitut, TU-Wien, Vienna, AUSTRIA
Hokkaido University, Sapporo, JAPAN

- I. Introduction: neutron as a particle/wave**
- II. Quantum Cheshire-cat & Pigeonhole effect**
- II. Uncertainty relations for quantum measurements**
- V. Summary**

Waves/Nonlocality in classical- and quantum-mechanics



Neutronen interferometry: quantum skier



The New Yorker Collection, Ch.Adams 1940

quarks
confinement radius
1.5 fm
virtual pions
gluon force
electromagnetic interaction
LATTICE PLANES
PERFECT SILICON CRYSTAL
Schrödinger cat-like part
smile of the wave function
 k / k_0
 x / Δ_c

The neutron

Particle

$$m = 1.674928(1) \times 10^{-27} \text{ kg}$$

$$s = \frac{1}{2} \hbar$$

$$\mu = -9.6491783(18) \times 10^{-27} \text{ J/T}$$

$$\tau = 887(2) \text{ s}$$

$$R = 0.7 \text{ fm}$$

$$\alpha = 12.0 (2.5) \times 10^{-4} \text{ fm}^3$$

u - d - d - quark structure

m ... mass, s ... spin, μ ... magnetic moment, τ ... β -decay lifetime, R ... (magnetic) confinement radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero

Feels four-forces

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m \cdot v}$$

Schrödinger

$$H\psi(\vec{r},t) = i\hbar \frac{\delta\psi(\vec{r},t)}{\delta t}$$

&

boundary conditions

Wave

$$\lambda_c = \frac{h}{m \cdot c} = 1.319695 (20) \times 10^{-15} \text{ m}$$

For thermal neutrons
 $\lambda = 2\text{Å}$, $v = 2\text{km/s}$, $E_{\text{kin}} = 20\text{meV}$

$$\lambda_B = \frac{h}{m \cdot v} = 1.8 \times 10^{-10} \text{ m}$$

$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8} \text{ m}$$

$$\Delta_p = v \cdot \Delta t \cong 10^{-2} \text{ m}$$

$$\Delta_d = v \cdot \tau = 1.942(5) \times 10^6 \text{ m}$$

$$0 \leq \chi \leq 2\pi (4\pi)$$

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk ... momentum width, Δt ... chopper opening time, v ... group velocity, χ ... phase.



Neutrons in quantum mechanics

Particle and wave properties

$$p = mv = h/\lambda$$

(L. De Broglie)

Schroedinger equation

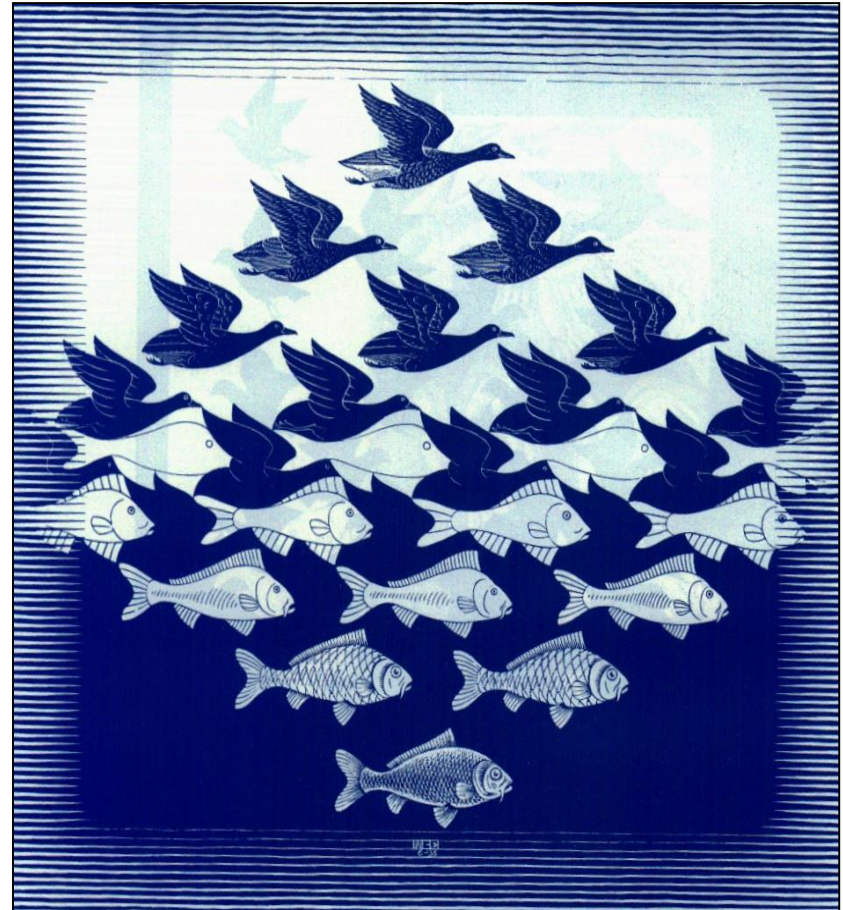
$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = H\Psi(\vec{r}, t)$$

(E. Schrödinger)

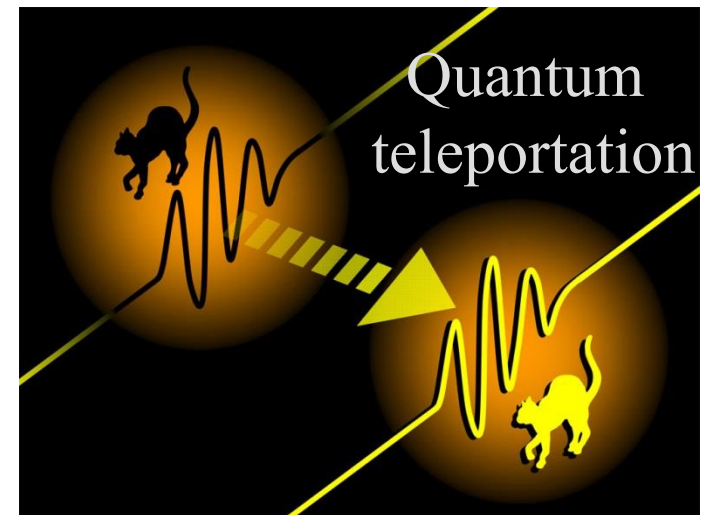
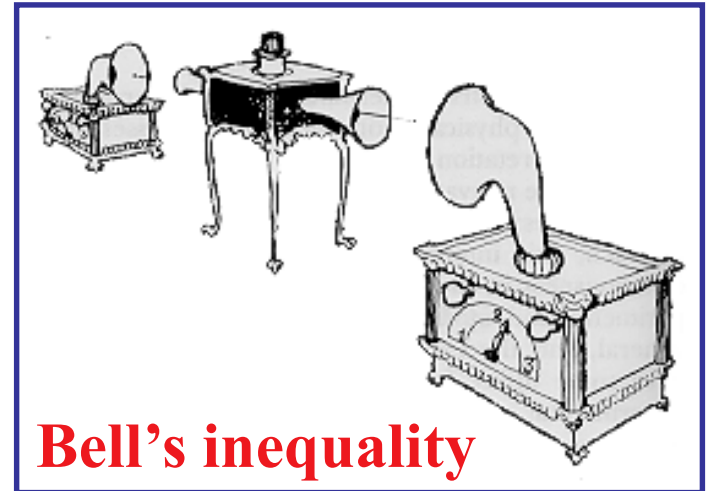
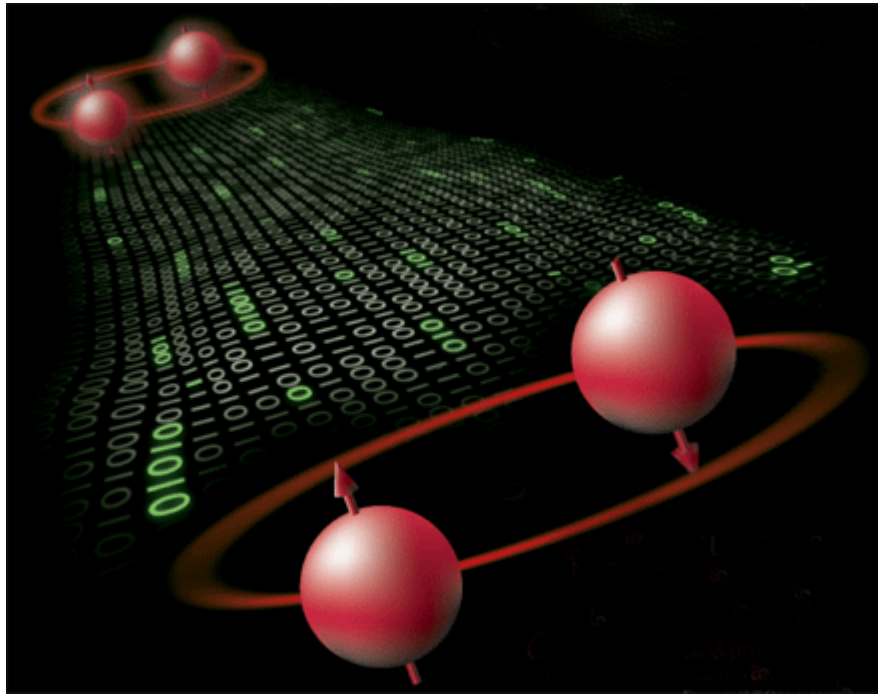
Uncertainty

$$\Delta x \Delta p \geq \hbar/2$$

(W. Heisenberg)



Quantum information technology: entanglement



Bi-partite and tri-partite entanglements

2-Particle Bell-State

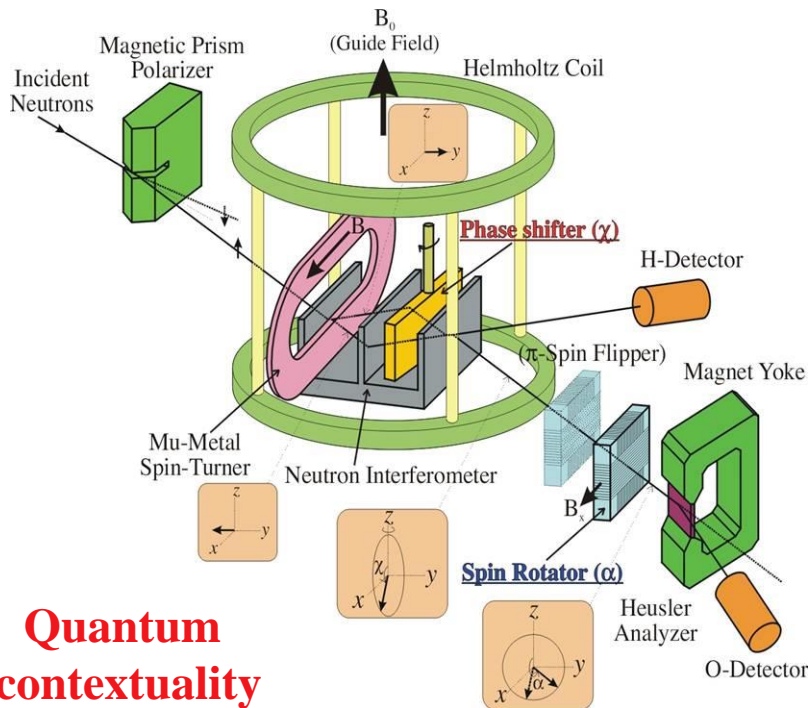
$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_I \otimes |\downarrow\rangle_{II} + |\downarrow\rangle_I \otimes |\uparrow\rangle_{II} \}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_s \otimes |I\rangle_p + |\downarrow\rangle_s \otimes |II\rangle_p \}$$

s, p represent 2-Spaces, e.g., spin & path



Quantum contextuality

Violation of Bell-like inequality

$$S' \equiv E'(\alpha_1, \chi_1) + E'(\alpha_1, \chi_2) - E'(\alpha_2, \chi_1) + E'(\alpha_2, \chi_2) = 2.051 \pm 0.019 > 2$$

Nature2003, NJP2011

Kochen-Specker-like contradiction

$$E_x \cdot E_y = 0.407 \xrightarrow{63\%} E' \equiv \langle \hat{X}_1 \hat{Y}_2 \cdot \hat{Y}_1 \hat{X}_2 \rangle = -0.861$$

PRL2006/2009

Tri-partite entanglement (GHZ-state)

$$|\Psi_{\text{Neutron}}\rangle = \{ |\Psi_I\rangle \otimes |\uparrow\rangle \otimes |\Psi(E_0)\rangle + (e^{i\chi} |\Psi_{II}\rangle) \otimes (e^{i\alpha} |\downarrow\rangle) \otimes (e^{i\gamma} |\Psi(E_0 + \hbar\omega_r)\rangle) \}$$

$$M_{\text{Measured}} = 2.558 \pm 0.004 > 2$$

PRA2010/NJP2013

Cheshire-cat



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Article Talk



Cheshire Cat

From Wikipedia, the free encyclopedia

This article is about

The Cheshire Cat (*Cheshire Cat*) is a character created by Lewis Carroll in *Alice's Adventures in Wonderland*. It is a distinctive mischievous

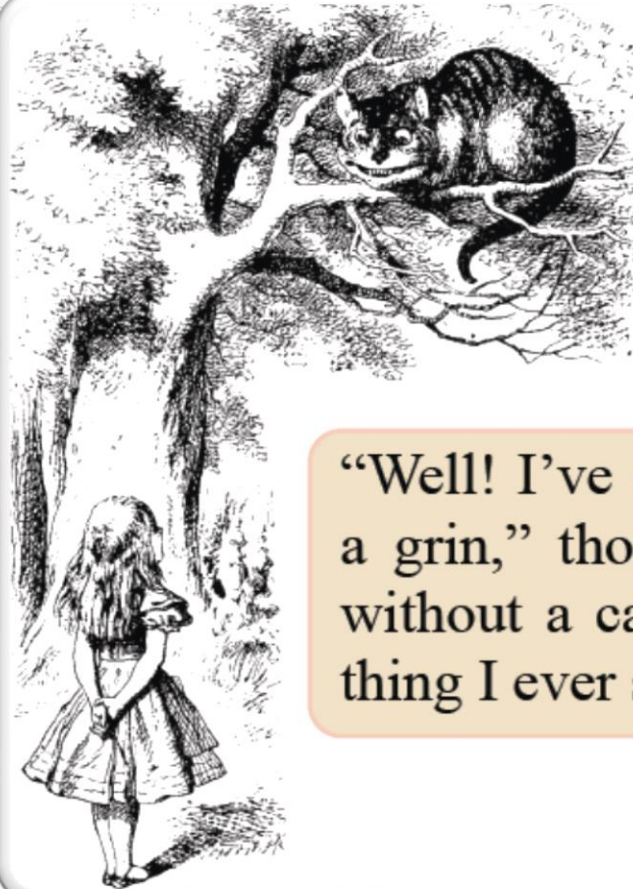
1 Origins

- 1.1 Dairy farming
- 1.2 Cheese mould
- 1.3 Church carving
- 1.4 Heraldic lion

2 Alice's Adventures in Wonderland

3 Cultural uses

- 3.1 Adaptations of the story
 - 3.1.1 Disney
 - 3.1.2 1999 TV series
 - 3.1.3 2010 film
- 3.2 Other major adaptations
- 3.3 Television
- 3.4 Anime and manga
- 3.5 Art
- 3.6 Businesses
- 3.7 Comics
- 3.8 Film
- 3.9 Games

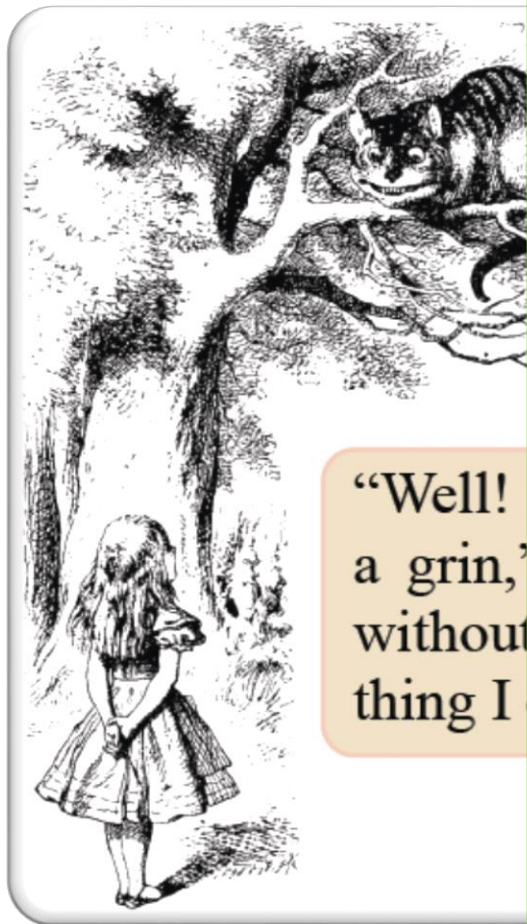


“Well! I’ve often seen a cat without a grin,” thought Alice; “but a grin without a cat! It’s the most curious thing I ever saw in all my life!”

First appearance *Alice's Adventures in Wonderland*

Created by Lewis Carroll

Quantum Cheshire-cat



New Journal of Physics

The open access journal for physics

Quantum Cheshire Cats

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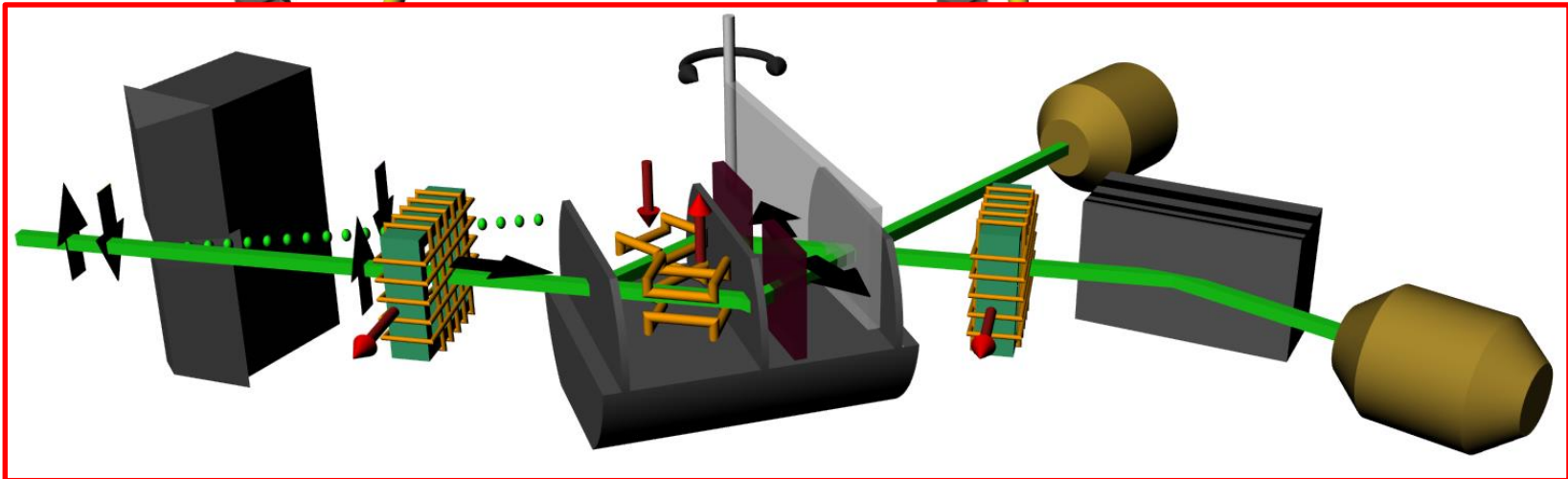
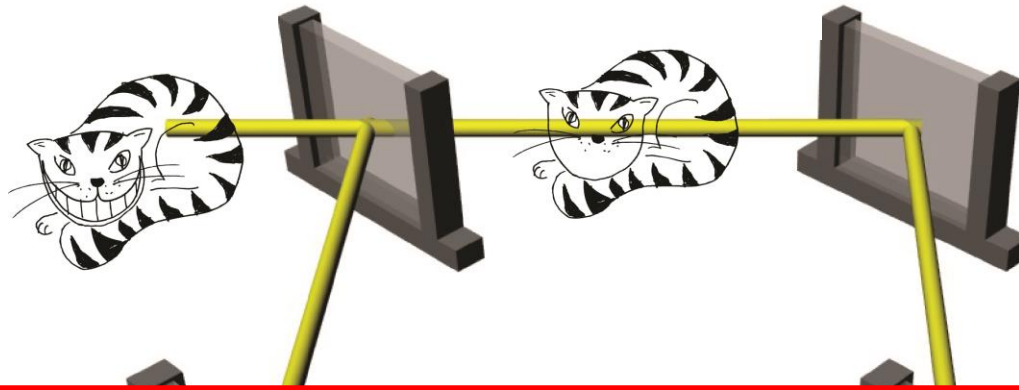
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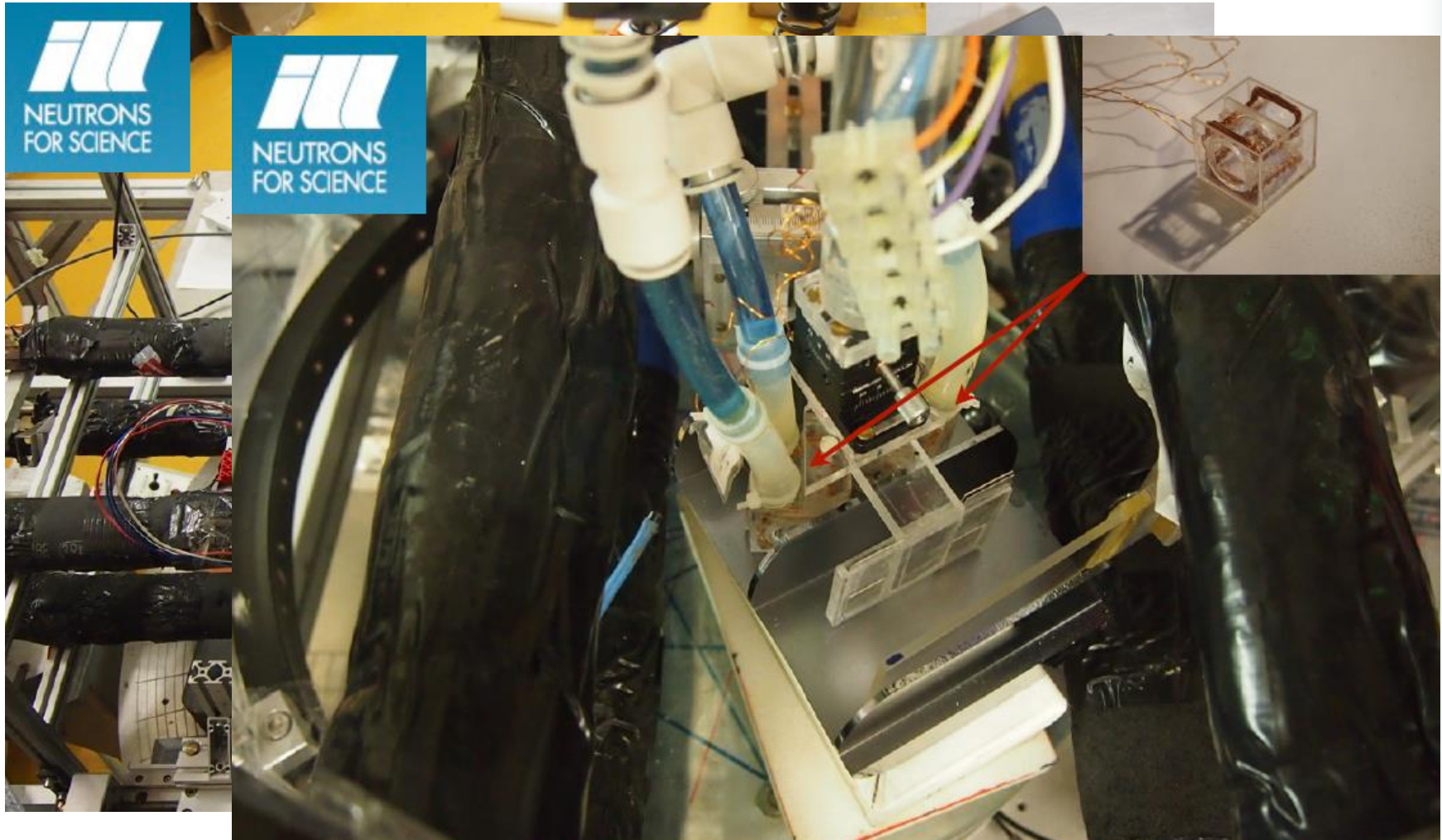
doi:10.1088/1367-2630/15/11/113015

Abstract. In this paper we present a quantum Cheshire Cat. In a pre- and post-selected experiment we find the Cat in one place, and its grin in another. The Cat is a photon, while the grin is its circular polarization.

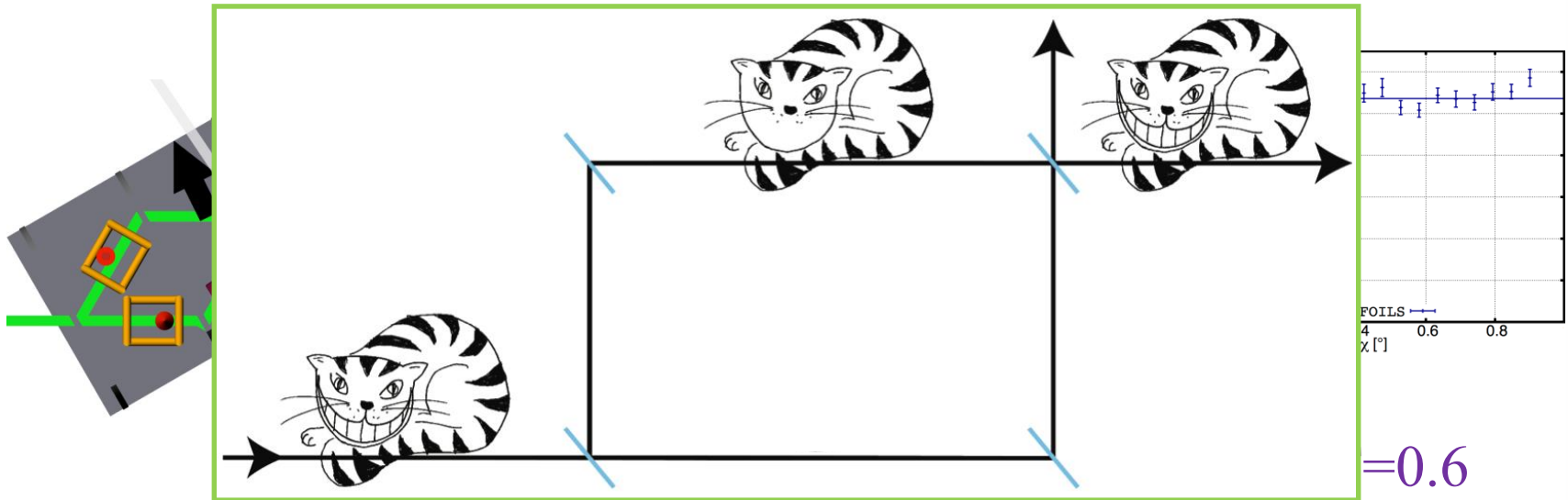
Quantum Cheshire-cat in neutron interferometer



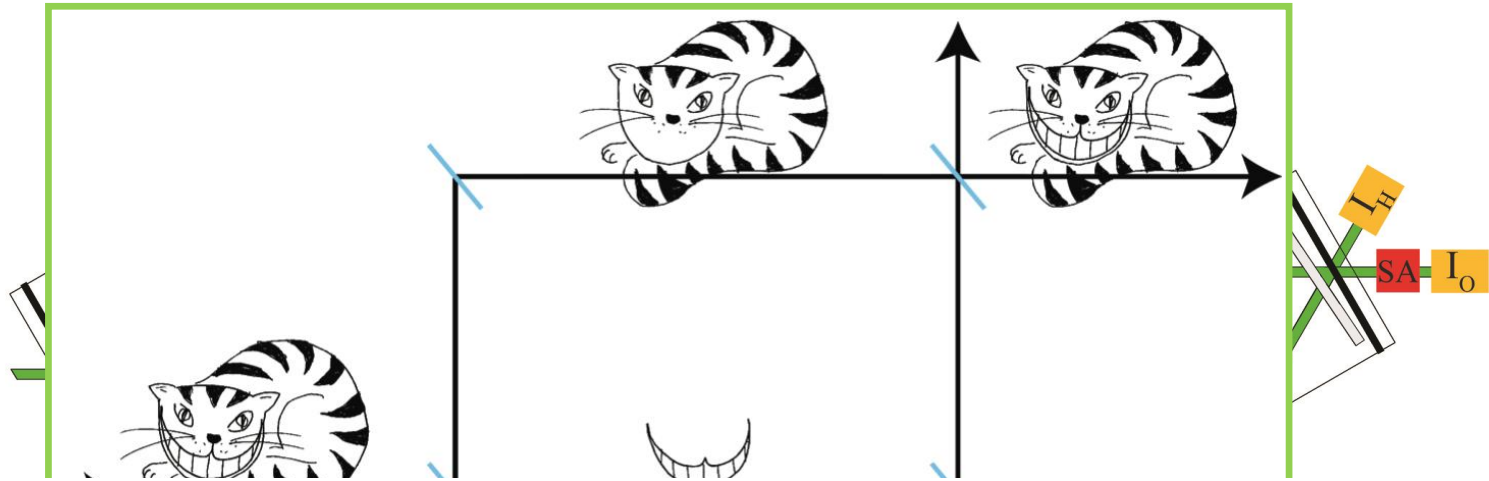
Quantum Cheshire-cat: experiment



Quantum Cheshire-cat: neutron(cat) in upper path



Quantum Cheshire-cat: spin(smile) in lower path

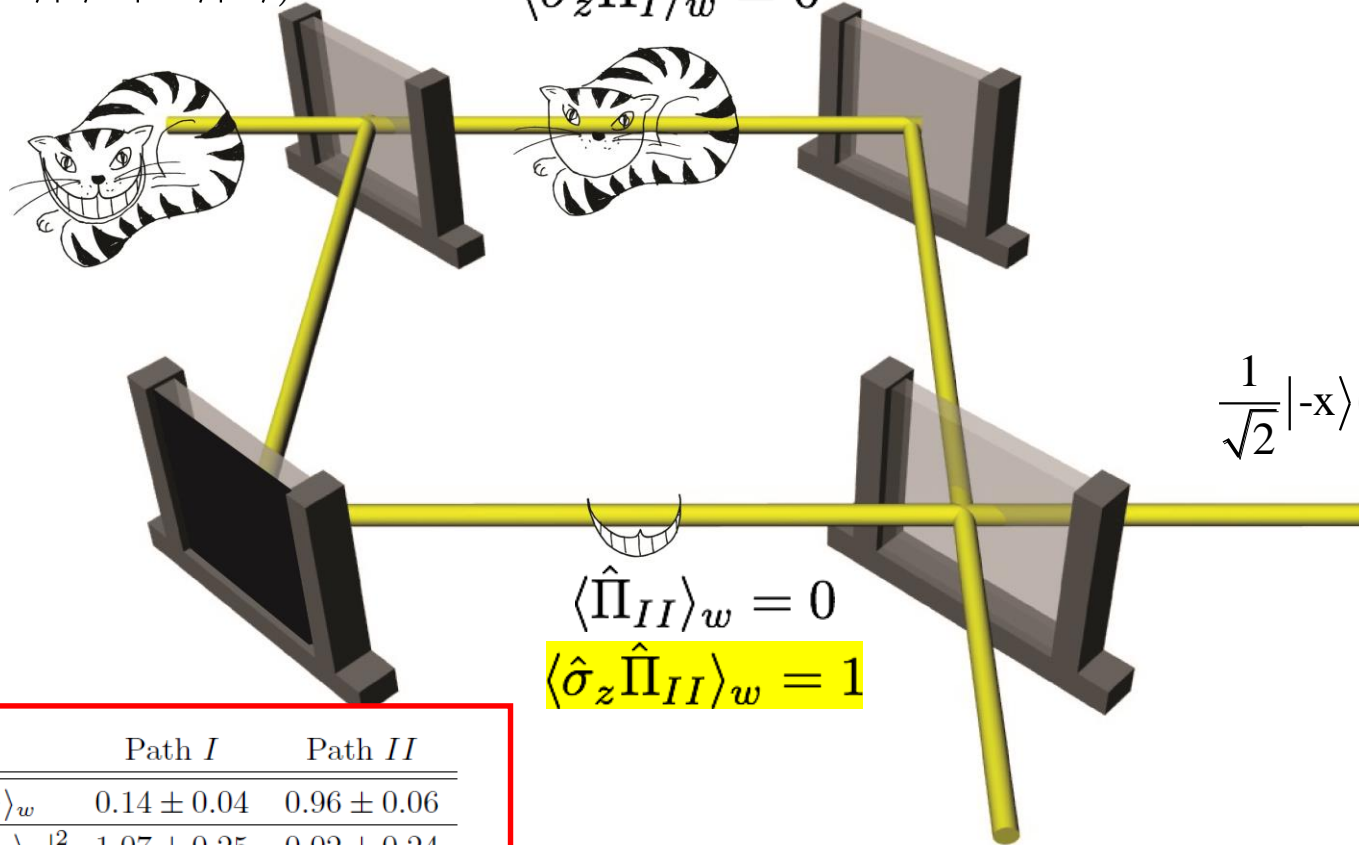


Quantum Cheshire-cat: final results

$$\frac{1}{\sqrt{2}}(|-x\rangle|I\rangle + |+x\rangle|II\rangle)$$

$$\langle \hat{\Pi}_I \rangle_w = 1$$

$$\langle \hat{\sigma}_z \hat{\Pi}_I \rangle_w = 0$$



$$\frac{1}{\sqrt{2}}|-x\rangle(|I\rangle + |II\rangle)$$

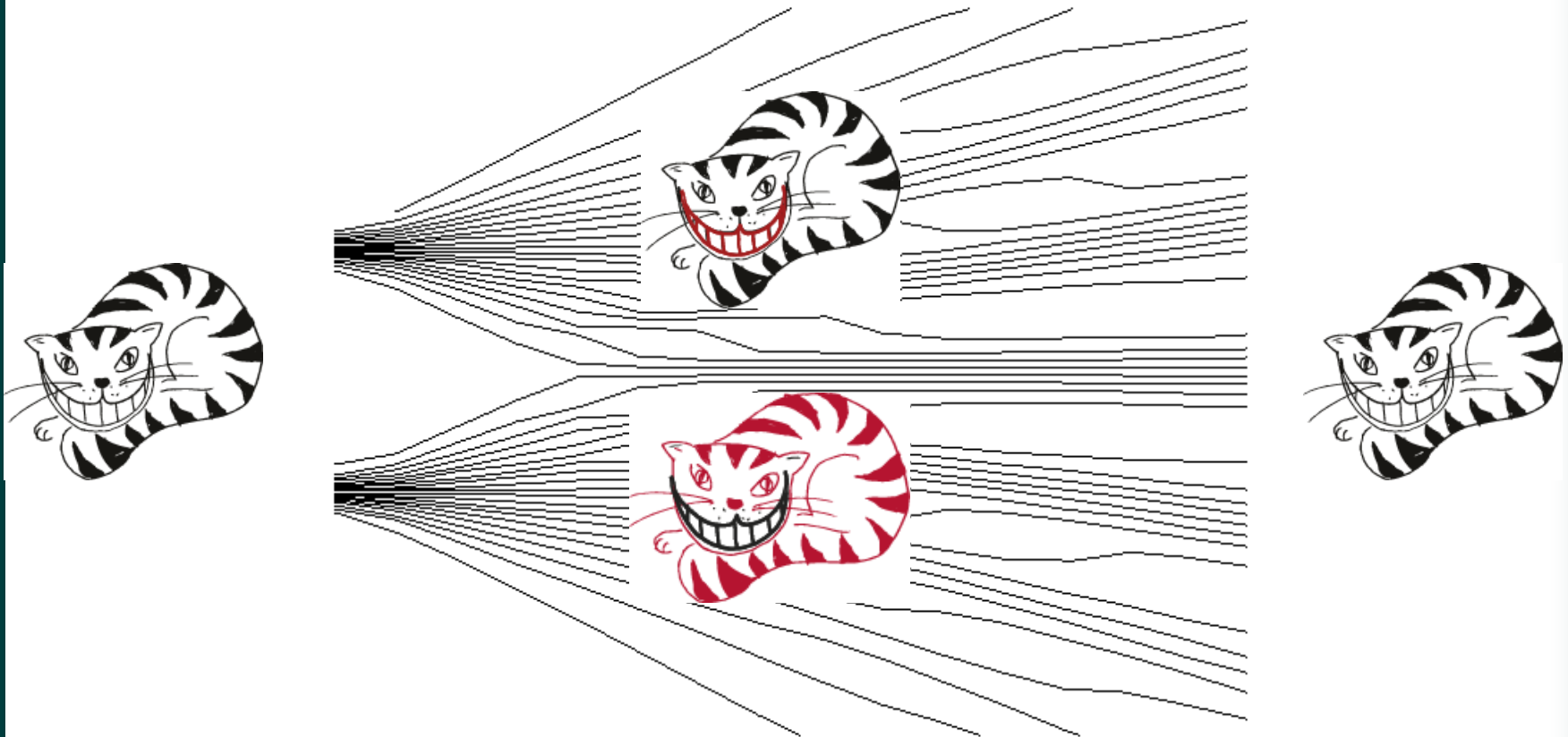
$$\langle \hat{\Pi}_{II} \rangle_w = 0$$

$$\langle \hat{\sigma}_z \hat{\Pi}_{II} \rangle_w = 1$$

	Path I	Path II
$\langle \hat{\Pi}_j \rangle_w$	0.14 ± 0.04	0.96 ± 0.06
$ \langle \hat{\sigma}_z \hat{\Pi}_j \rangle_w ^2$	1.07 ± 0.25	0.02 ± 0.24

T. Denkmayr et al. Nature Comm. 5:4492 (2014).

Quantum Cheshire-cat: invisible cat/spin ???



<http://Bohmian-mechanics.net/>

Reactions



29 July 2014 Last updated

'Quantum'

By James Morgan
Science reporter, BBC



The Cheshire Cat myst...

DER STANDARD

FORSCHUNG SPEZIAL

MITTWOCH, 30. JULI 2014

Die Katze verschwindet, ihr Grinsen bleibt

Die Presse SAMSTAG, 16. AUGUST 2014

Wiener Experiment

Wien – Die Quantenmechanik des Menschen schon ein konzeptionelles Paradoxon

Wie Krankheiten den Lauf der Geschichte ändern Seite 54

Versteckte Emissionen eines lang verbotenen Treibhausgas Seite 55

Eine überprüfte Qualitätskontrolle, die zu Muskelschwund führt Seite 55

Ein alter Stern war Zeuge einer heftigen Explosion Seite 55

Von grinsenden Katzen und nackten Neutronen

Mithilfe schwacher Messungen schaffen Forscher im Labor das Analogon einer Cheshire Cat

Jeder Physikstudent lernt, dass es unmöglich ist, Informationen über ein Quantenobjekt zu gewinnen, ohne es zu stören. Doch mit schwachen Messungen wird das Unmögliche möglich. Bei der Interpretation der Resultate ist allerdings Vorsicht angebracht.

Christian Speiser

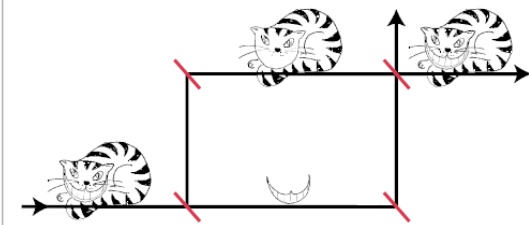
Quantenphysiker haben eine Schwäche für Katzen. Ein berühmtes Beispiel ist Schrödingers Katze. Dabei handelt es sich um ein imaginäres Wesen, das zusammen mit einer todbringenden radioaktiven Substanz in einer Kiste eingesperrt ist. Laut Quantentheorie schwebt die Katze in einem Zustand zwischen tot und lebendig, bis eine Messung Gewissheit darüber bringt, ob ein radioaktiver Zerfall stattgefunden hat oder nicht. Ganz nach dem Geschmack der Quantenphysiker ist auch die Cheshire Cat aus dem Kinderbuch „Alice im Wunderland“. Die kleine Alice begegnet dort einer Katze, aber die sie sich sehr wundert: „Oh, ich habe oft eine Katze ohne Grinsen gesehen, aber ein Grinsen ohne Katze, so etwas Merkwürdiges habe ich in meinem Leben noch nicht gesehen.“

Getrennte Wege

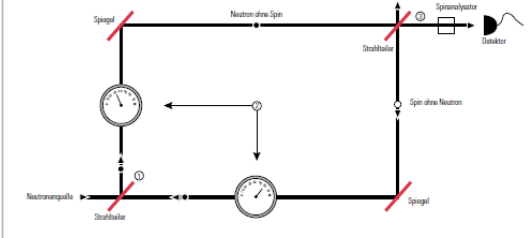
Wer glaubt, so etwas Absurdes wie ein Grinsen ohne Katze gebe es nur in der Fabel, der irrt. Am Institut Laue-Langevin in Grenoble ist es österreichischen, französischen und amerikanischen Forschern kürzlich gelungen, die quantenmechanische Entsprechung einer Cheshire Cat zu beobachten. In einem Interferometer konnte die Gruppe um Tobias Denkmayr und Yuji Hasegawa vom Atominstitut der TU Wien Neutronen von ihren magnetischen Momenten trennen. Während die Neutronen auf einen Pfad des Interferometers einschlugen, wählten ihre magnetischen Momente den anderen. Das Grinsen der Katzen hatte sich also gewissermaßen vernetztändig (siehe Grafik).

Das Neutronen-Experiment lässt aufhorchen, weil es auf einen un-

Die Cheshire-Katze und ihre quantenmechanische Entsprechung



- 1. Fotoelektron: Die Neutronen werden so präpariert, dass die Spin im oberen Pfad in Bewegungsrichtung und im unteren Pfad in die entgegengesetzte Richtung zeigt.
- 2. Schwache Messung: Um die Neutronenpopulation in den Pfaden zu messen, wird ein schwach absorbierender Filter verwendet. Ein erster Pfad dieser schwachen Messung können Einfluss auf die Zahl der polarisierbaren Neutronen, im anderen schon. Das Ergebnis nur getrennter Cheshire Katze ist offensichtlich.
- 3. Photoelektron: Nach der Zusammenführung der beiden Strahlen erfolgt die Nachweise der Neutronen. Dabei werden nur jene Neutronen gezählt, deren Spin in Bewegungsrichtung zeigt. Nur für diese polarisierbaren Neutronen gilt die Aussage, dass die Neutronen und ihr Spin getrenntes Wege gehen.



Größe kennt, desto grösser wird die Unsicherheit bezüglich der anderen. Für Aufsehen sorgte 2011 auch eine Arbeit von Aephraim Steinberg und sei-

Lust auf Salz

Schadet ein geringer Salzkonsum?

Die Frage, wie viel Salz man maximal verzehren sollte, erlirbt seit langem die Gemüter. Öl ins Feuer giessen nun die Initiatoren einer weltumspannenden Bevölkerungsstudie.

Nicola von Lutterotti

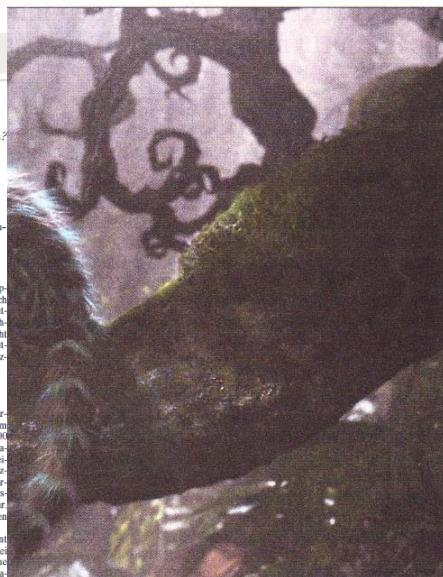
Laut den gängigen medizinischen Empfehlungen sollten Erwachsene täglich höchstens 2 Gramm Natrium, das entspricht etwa 5 Gramm Salz, zu sich nehmen. Jenseits dieser Menge besteht demnach die Gefahr, dass der Blutdruck und damit das Risiko für Herzleiden und Schlaganfälle ansteigt.

Unerwartetes Ergebnis

In eine andere Richtung weisen die Ergebnisse einer neuen Studie mit dem Kürzel „PURE“, an der mehr als 100.000 Personen aus 17 Ländern beteiligt waren. Wie sie nämlich nahelegt, erleiden Personen mit einem täglichen Salzkonsum von maximal 5 Gramm eher Erkrankungen des Herz-Kreislauf-Systems als solche mit höherem Verzehr. Müssen die medizinischen Leitlinien also wieder umgeschrieben werden?

Das wir auf Salziges stehen, kommt nicht von ungefähr. Natrium spielt bei etlichen Stoffwechselprozessen eine wichtige Rolle. Der menschliche Organismus ist daher seit Urzeiten darauf geübt, das lebenswichtige Mineral aufzunehmen und im Körper zu halten. Angesichts dieser evolutionären Weichenstellung ist wenig erstaunlich, dass mehr als 90 Prozent der Teilnehmer von PURE mehr, teilweise sogar deutlich mehr Salz verzehrten als gemeinsam empfohlen.

Der durchschnittliche Salzkonsum betrug rund 12 Gramm am Tag. Wie sich zeigte, erlitten im Verlauf von knapp vier Jahren 317 Probanden eine teilweise tödliche Herz-Kreislauf-Attacke. Am seltensten betroffen waren hiervon Personen, die täglich etwa 41 Gramm Natrium ausschieden, also rund 10 Gramm Salz am Tag verzehrten. Sowohl oberhalb als auch unterhalb dieses Schwellenwerts nahm die Erkrankungsfahrde deutlich zu. In beiden Richtungen gab dabei je weiter sich der Natriumgehalt im Urin von 4 Gramm



...stier der Quantenphysiker.

[NG Collect/Interfoto/picturesdesk.com]

Alice im Wunderland

von ihren Eigenschaften zu trennen. Im Wunderland“.



Quantum pigeonhole effect 1

Quantum violation of the pigeonhole principle and the nature of quantum correlations

Yakir Aharonov^{a,b,c,1}, Fabrizio Colombo^d, Sandu Popescu^{c,e}, Irene Sabadini^d, Daniele C. Struppa^{b,c}, and Jeff Tollaksen^{b,c}

^aSchool of
92866; ^cIns
and ^eH. H.

Contributed
532-535

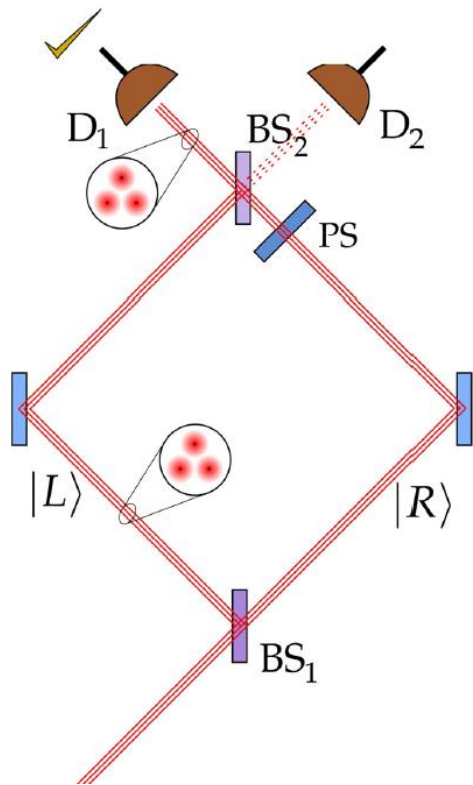
The pigeonhole principle: "If you put three pigeons in two pigeonholes, at least two of the pigeons end up in the same hole," is an obvious yet fundamental principle of nature as it captures the very essence of counting. Here however we show that in quantum mechanics this is not true! We find instances when three quantum particles are put in two boxes, yet no two particles are in the same box. Furthermore, we show that the above "quantum pigeonhole principle" is only one of a host of related quantum effects, and points to a very interesting structure of quantum mechanics that was hitherto unnoticed. Our results shed new light on the very notions of separability and correlations in quantum mechanics and on the nature of interactions. It also presents a new role for entanglement, complementary to the usual one. Finally, interferometric experiments that illustrate our effects are proposed.

Orange, CA
Milan, Italy;

s.1522411112

Quantum pigeonhole effect 2

$$|\Phi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad \text{where } |\pm i\rangle \equiv \frac{1}{\sqrt{2}} \{|L\rangle \pm i|R\rangle\}$$



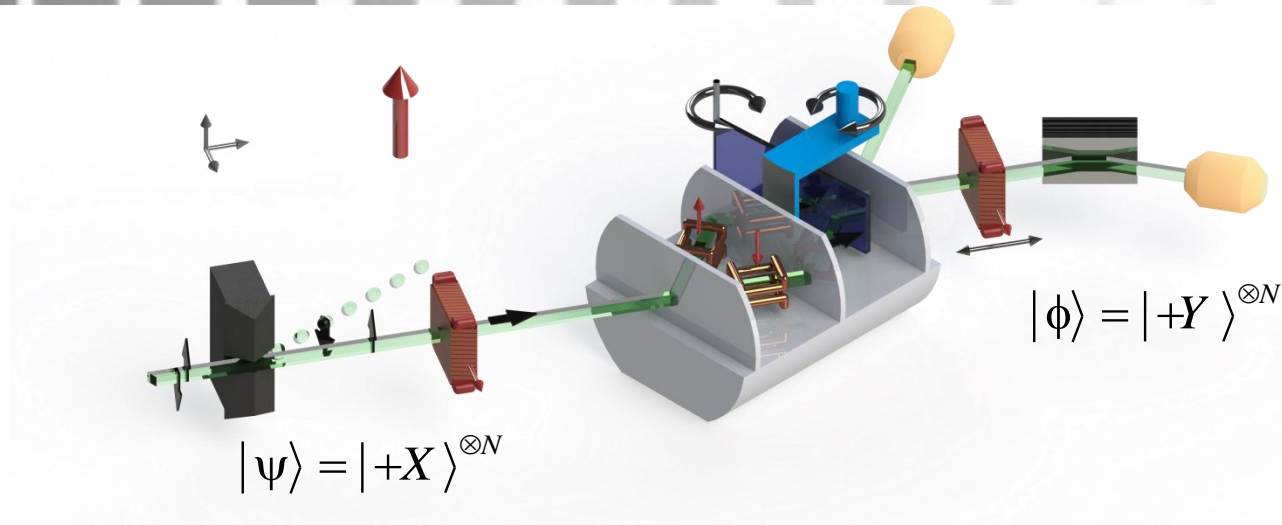
$$\begin{cases} \hat{\Pi}_{12}^{same} = \hat{\Pi}_{12}^{LL} + \hat{\Pi}_{12}^{RR} \\ \hat{\Pi}_{12}^{diff} = \hat{\Pi}_{12}^{LR} + \hat{\Pi}_{12}^{RL} \end{cases} \quad \text{where } \hat{\Pi}_{12}^{LL} \equiv |L\rangle\langle L| \text{ etc.}$$

$$\begin{aligned} & \langle \Phi | \hat{\Pi}_{12}^{same} | \Psi \rangle \\ & \propto \left({}_1\langle L | - i {}_1\langle R | \right) \left({}_2\langle L | - i {}_2\langle R | \right) {}_3\langle + | \\ & \quad \times \left\{ \left(|L\rangle_1 |L\rangle_2 + |R\rangle_1 |R\rangle_2 \right) |+\rangle_3 \right\} \\ & = \left({}_1\langle L | {}_2\langle L | - i {}_1\langle L | {}_2\langle R | - i {}_1\langle R | {}_2\langle L | - {}_1\langle R | {}_2\langle R | \right) \\ & \quad \times \left(|L\rangle_1 |L\rangle_2 + |R\rangle_1 |R\rangle_2 \right) \\ & = 0 \quad \text{No particles are in the same path!} \end{aligned}$$

$$|\Psi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad \text{where } |\pm\rangle \equiv \frac{1}{\sqrt{2}} \{|L\rangle \pm |R\rangle\}$$

Y. Aharonov et al. PNAS 113, 532 (2016).

Quantum pigeonhole effect in neutron interferometer



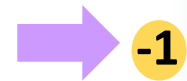
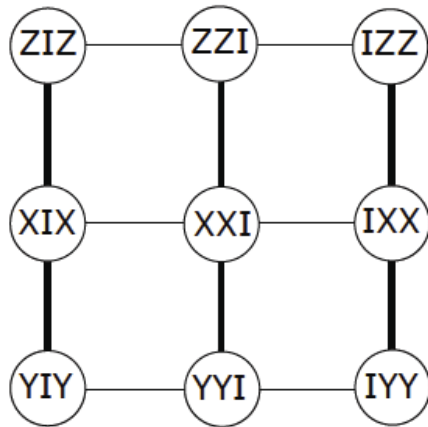
$$\begin{cases} \hat{\Pi}^{even} = \hat{\Pi}_+ \hat{\Pi}_+ + \hat{\Pi}_- \hat{\Pi}_- \\ \hat{\Pi}^{odd} = \hat{\Pi}_+ \hat{\Pi}_- + \hat{\Pi}_- \hat{\Pi}_+ \end{cases} \quad \text{where } \hat{\Pi}_{\pm} \equiv |\pm Z\rangle\langle \pm Z|$$

$$\hat{\Pi}_w^{even} = 0 \ \& \ \hat{\Pi}_w^{odd} = 1, \ \text{then} \ \left(\hat{\sigma}_z^{path} \bullet \hat{\sigma}_z^{path} \right)_w = -1$$

No two pigeons ever seem to be in the same box!!!

Quantum contextuality in neutron interferometer

(a) Magic square



$$\Pi_1^{(3)} = | +Z, +Z, +Z \rangle \langle +Z, +Z, +Z | + | -Z, -Z, -Z \rangle \langle -Z, -Z, -Z |,$$

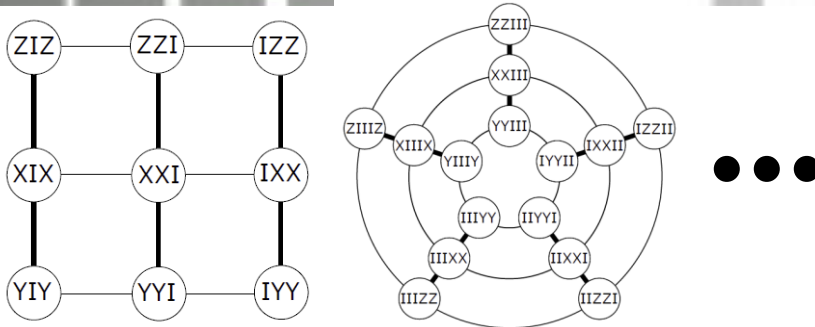
$$\Pi_2^{(3)} = | +Z, -Z, +Z \rangle \langle +Z, -Z, +Z | + | -Z, +Z, -Z \rangle \langle -Z, +Z, -Z |,$$

$$\Pi_3^{(3)} = | -Z, +Z, +Z \rangle \langle -Z, +Z, +Z | + | +Z, -Z, -Z \rangle \langle +Z, -Z, -Z |,$$

$$\Pi_4^{(3)} = | -Z, -Z, +Z \rangle \langle -Z, -Z, +Z | + | +Z, +Z, -Z \rangle \langle +Z, +Z, -Z |,$$

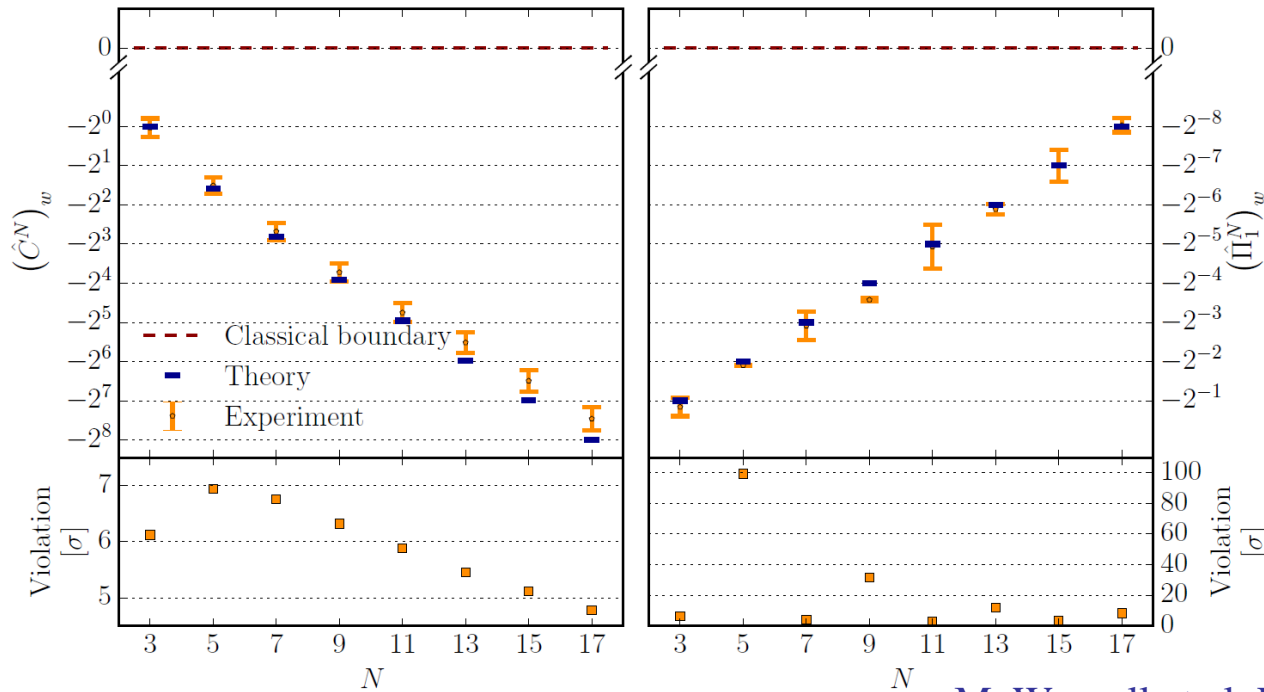
$$(\Pi_1^{(3)})_w = \prod_{n=1}^3 \frac{1 + Z_w}{2} + \prod_{n=1}^3 \frac{1 - Z_w}{2} = -\frac{1}{2} \notin [0, 1]$$

Quantum contextuality in neutron interferometer



$$(\Pi_i^{(N)})_w = \prod_{n=1}^N \frac{1 + (-1)^{x_{i,n}^{(N)}} Z_w}{2} + \prod_{n=1}^N \frac{1 - (-1)^{x_{i,n}^{(N)}} Z_w}{2}$$

$$C^{(N)} = \text{Re} \left(I - \sum_{i=1}^{2^N-1} s_i (\Pi_i^{(N)})_w \right)$$



M. Waegell et al. PRA 96, 052131 (2017).

Weak values via weak/strong measurements

PRL 116, 040502 (2016)

PHYSICAL REVIEW LETTERS

week ending
29 JANUARY 2016

Strong Measurements Give a Better Direct Measurement of the Quantum Wave Function

Giuseppe Vallone and Daniele Dequal

Department of Information Engineering, University of Padova, via Gradenigo 6/B, 35131 Padova, Italy
(Received 24 April 2015; revised manuscript received 24 December 2015; published 29 January 2016)

Weak measurements have thus far been considered instrumental in the so-called direct measurement of the quantum wave function [J. S. Lundeen, *Nature (London)* 474, 188 (2011)]. Here we show that a direct measurement of the wave function can be obtained by using **measurements of arbitrary strength**. In particular, in the case of strong measurements, i.e., those in which the coupling between the system and the measuring apparatus is maximum, we compared **the precision and the accuracy** of the two methods, by showing that **strong measurements outperform weak measurements** in both for arbitrary quantum states in most cases. We also give the exact expression of the difference between the original and reconstructed

Accuracy of DWT.—In the case of DWT, the obtained wave function $\psi_{W,x}$ is an approximation of the correct wave function ψ_x . We now evaluate the accuracy of the DWT, namely, the errors arising by using Eq. (3) in place of the exact values of (4). As done in Ref. [13], we define the accuracy in terms of the trace distance \mathcal{D} between the correct wave function ψ_x and the weak-value approximation $\psi_{W,x}$ [19], that for pure states reduces to $\mathcal{D} = \sqrt{1 - |\langle \psi | \psi_W \rangle|^2}$. We first give the analytical expression of \mathcal{D} in terms of the original wave function and then show how \mathcal{D} can be upper bounded by using the measurement outcomes.

Precision of the DWT.—An important performance parameter is the precision of the method, namely, the statistical errors on the estimated wave function. In particular, it is important to evaluate the scaling of such errors with the number of measurements. To this purpose, we evaluated the *mean square statistical error* $\delta\psi$ of the DWT and DST methods, obtained by summing the squares of the statistical error on the different ψ_x :

$$\delta\psi = \sqrt{\sum_x |\delta\psi_x|^2}. \quad (9)$$

WV via weak/strong measurements: experiment

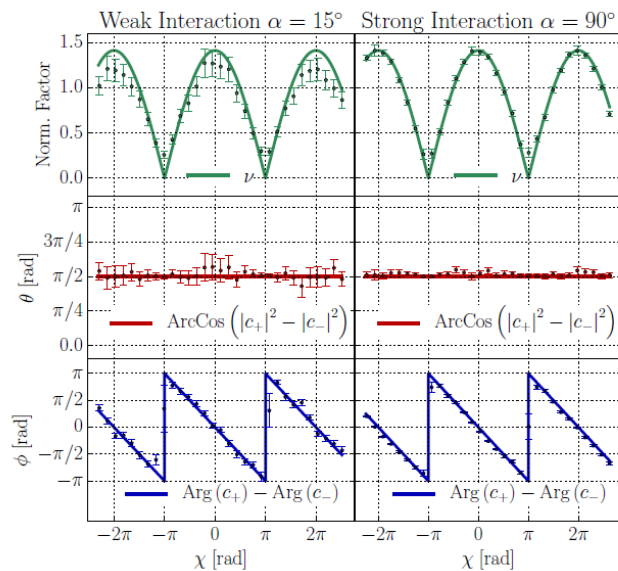
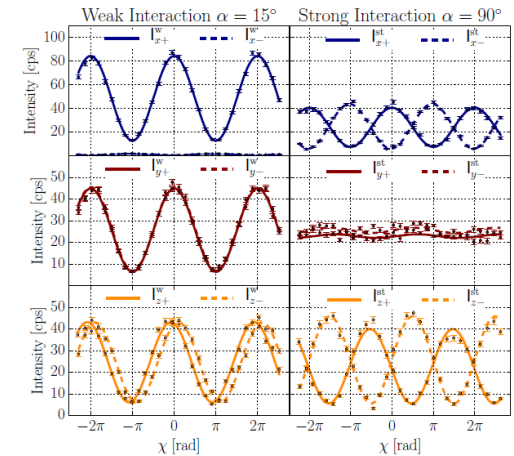
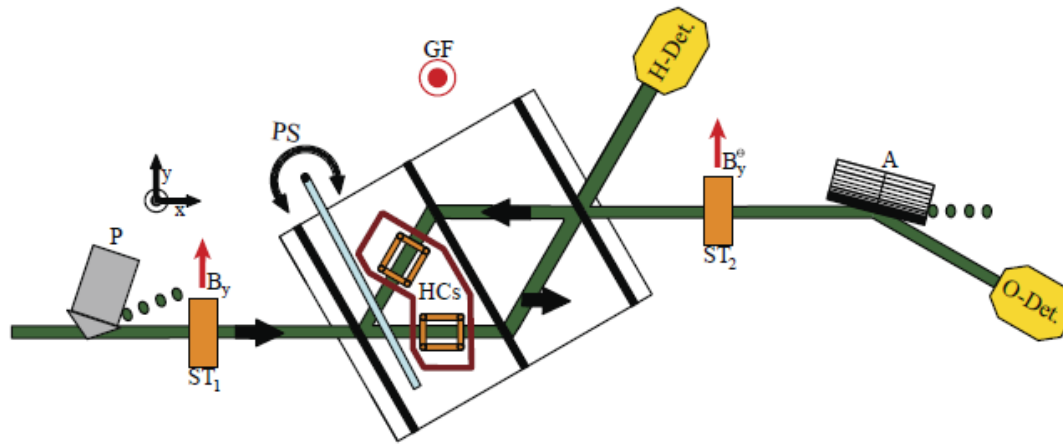


TABLE I. Quantitative comparison of precision $\bar{\sigma}$ and accuracy $\bar{\Delta}$ of the weak and the strong interaction approach.

Precision $\bar{\sigma}$		Accuracy $\bar{\Delta}$			
Weak	Strong	Weak	Strong		
ν	0.100	0.036	ν	0.152	0.062
θ	0.191	0.065	θ	0.100	0.067
ϕ	0.355	0.159	ϕ	0.860	0.580

T. Denkmayr et al. PRL **118**, 010402 (2017).

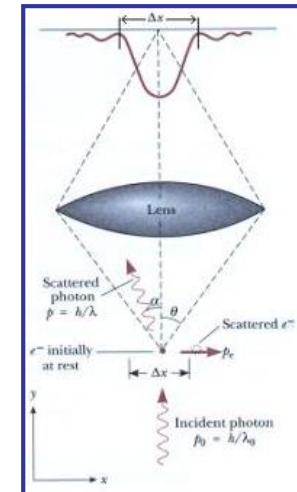
Uncertainty relation: historical

- In 1927 Heisenberg postulated an uncertainty principle:

γ -ray thought experiment

$$\rightarrow p_1 q_1 \approx h$$

with q_1 (mean error) & p_1 (discontinuous change)



- Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die un stetige Änderung von p beim Compton-effekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

$$p_1 q_1 \sim h. \quad (1)$$

Ozawa's Universally Valid Uncertainty Relation

PHYSICAL REVIEW A 67, 042105 (2003)

Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement

Masanao Ozawa

Graduate School of Information Sciences, Tohoku University, Aoba-ku, Sendai, 980-8579, Japan

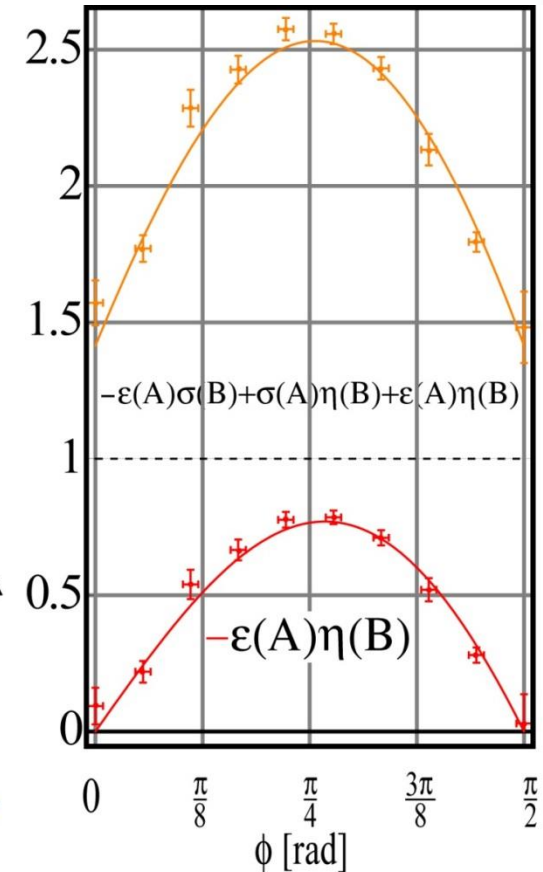
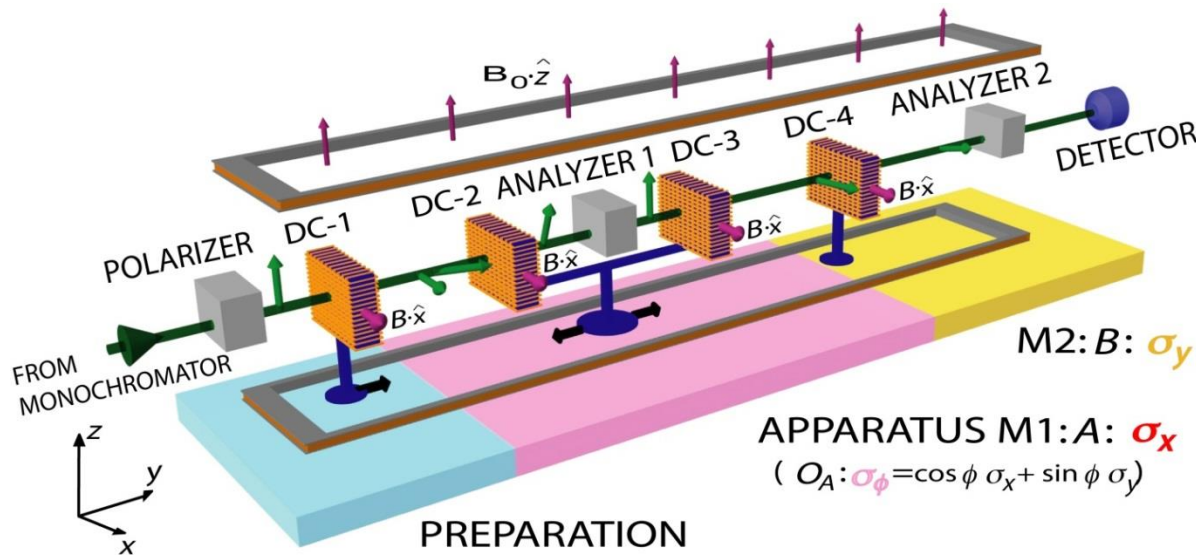
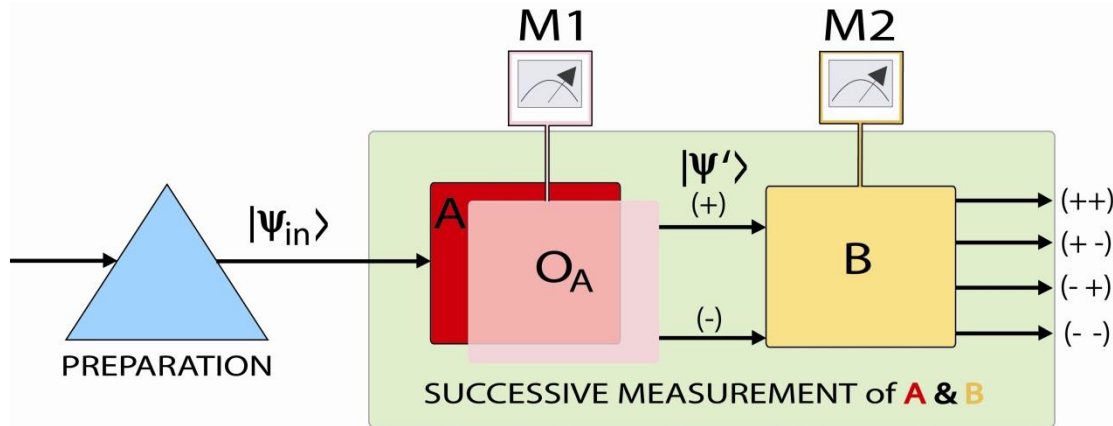
(Received 9 October 2002; published 11 April 2003)

The Heisenberg uncertainty principle states that the product of the noise in a position measurement and the momentum disturbance caused by that measurement should be no less than the limit set by Planck's constant $\hbar/2$ as demonstrated by Heisenberg's thought experiment using a γ -ray microscope. Here it is shown that this common assumption is not universally true: a universally valid trade-off relation between the noise and the disturbance has an additional correlation term, which is redundant when the intervention brought by the

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

- rigorous theoretical treatments of quantum measurements:
 - **first term:** error of the first measurement, disturbance on the second measurement
 - **second and third terms:** crosstalks between spreads of wavefunctions and error/disturbance

Experimental test



J. Erhart et al.,
Nature Phys. 8, 185-189 (2012)

Publications by other groups

PRL **109**, 100404 (2012)

PHYSICAL REVIEW LETTERS

week ending
7 SEPTEMBER 2012



Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aephraim M. Steinberg

PRL **110**, 220402 (2013)

PHYSICAL REVIEW LETTERS

week ending
31 MAY 2013

Experimental Test of Universal Complementarity Relations

Morgan M. Weston, Michael J. W. Hall, Matthew S. Palsson, Howard M. Wiseman, and Cyril Branciard

ArXiv: 1304.2071

Centre for Quantum

How well can one jointly measure two incompatible observables on a given quantum state?

(Received

Cyril Branciard

Centre for Engineered Quantum Systems and School of Physics,
The University of Queensland, St Lucia, Queensland 4072, Australia

(Dated: April 9, 2013)

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DOI: 10.1103/PhysRevLett.110.220402

Heisenberg's uncertainty principle is one of the main tenets of quantum mechanics, and its fundamental importance for our understanding of quantum foundations is widely appreciated. However, its interpretation: although Heisenberg's first argument was that the measurement of one observable necessarily disturbs another incompatible observable, standard quantum mechanics predicts the indeterminacy of the outcomes when either one or the other observable is measured, precisely Heisenberg's intuition. Even if two incompatible observables are measured, they still approximate their joint measurement, at the price of introducing a disturbance to the measurement of each of them. We present a new, tight relation between the error on one observable versus the error on the other. As a consequence, we characterize the disturbance of an observable induced by the measurement of another, and derive a stronger error-disturbance relation for this scenario.

OPEN Experimental violation and reformulation of the Heisenberg's error-disturbance uncertainty relation

SUBJECT AREAS:
QUANTUM MECHANICS
QUANTUM METROLOGY
QUANTUM INFORMATION
QUANTUM OPTICS

Received 7 August 2012
Accepted 2 July 2013
Published 17 July 2013

So-Young Baek^{1*}, Fumihiko Kaneda¹, Masanao Ozawa² & Keiichi Edamatsu¹

¹Research Institute of Electrical Communication, Tohoku University, Sendai 980-8577, Japan, ²Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan.

The uncertainty principle formulated by Heisenberg in 1927 describes a trade-off between the error of a measurement of one observable and the disturbance caused on another complementary observable such that their product should be no less than the limit set by Planck's constant. However, Ozawa in 1988 showed a model of position measurement that breaks Heisenberg's relation and in 2003 revealed an alternative relation for error and disturbance to be proven universally valid. Here, we report an experimental test of Ozawa's relation for a single-photon polarization qubit, exploiting a more general class of quantum measurements than the class of projective measurements. The test is carried out by linear optical devices and realizes an indirect measurement model that breaks Heisenberg's relation throughout the range of our experimental parameter and yet validates Ozawa's relation.

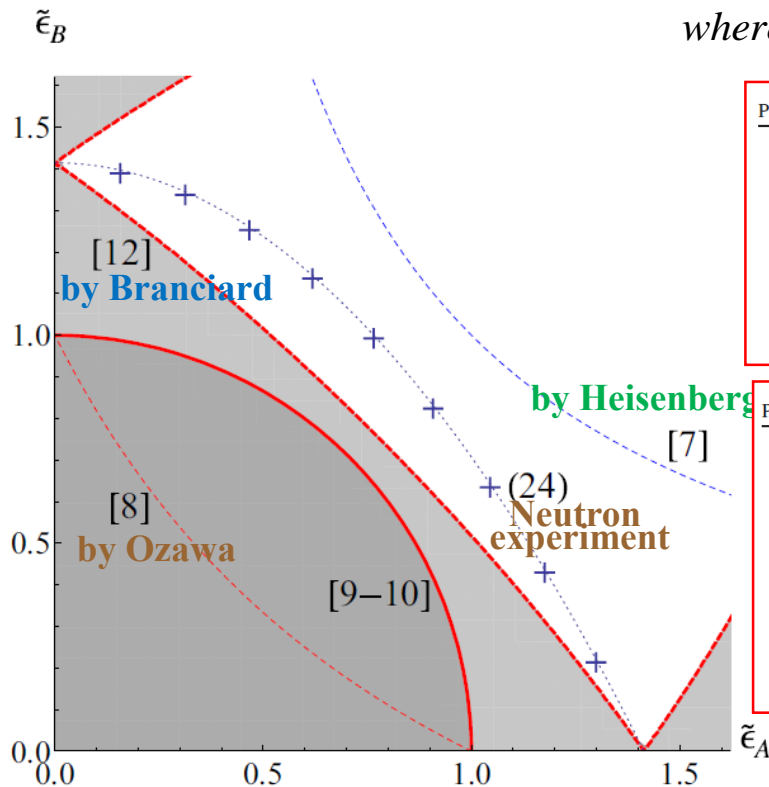
Correspondence and requests for materials to C.B. (cyril.branciard@uq.edu.au)



Tight relation derived by Branciard

$$\left[2\tilde{\varepsilon}(A)\tilde{\eta}(B)\sqrt{1-C^2} + \tilde{\varepsilon}(A)^2 + \tilde{\eta}(B)^2 \right]^{1/2} \geq C,$$

where $\tilde{\varepsilon} = \varepsilon\sqrt{1-\varepsilon^2/4}$, $\tilde{\eta} = \eta\sqrt{1-\eta^2/4}$, $C \equiv |\langle \psi | [A, B] | \psi \rangle| / 2$



PRL 112, 020401 (2014) PHYSICAL REVIEW LETTERS

Experimental Joint Quantum Measurements with Minimum Error

Martin Ringbauer,^{1,2,*} Devon N. Biggerstaff,^{1,2} Matthew A. Broome,^{1,2} Aleksandra Fedotina,^{1,2} Cyril Branciard,¹ and Andrew G. White^{1,2}

¹Centre for Engineered Quantum Systems, School of Mathematics and Physics, University of Queensland, Brisbane QLD 4072, Australia
²Centre for Quantum Computer and Communication Technology, School of Mathematics, University of Queensland, Brisbane QLD 4072, Australia
 (Received 27 August 2013; published 15 January 2014)

errors $\varepsilon_A, \varepsilon_B$. Contrary to the Heisenberg-Branciard relation [7] (dashed)

PRL 112, 020402 (2014) PHYSICAL REVIEW LETTERS

Experimental Test of Error-Disturbance Uncertainty Relation

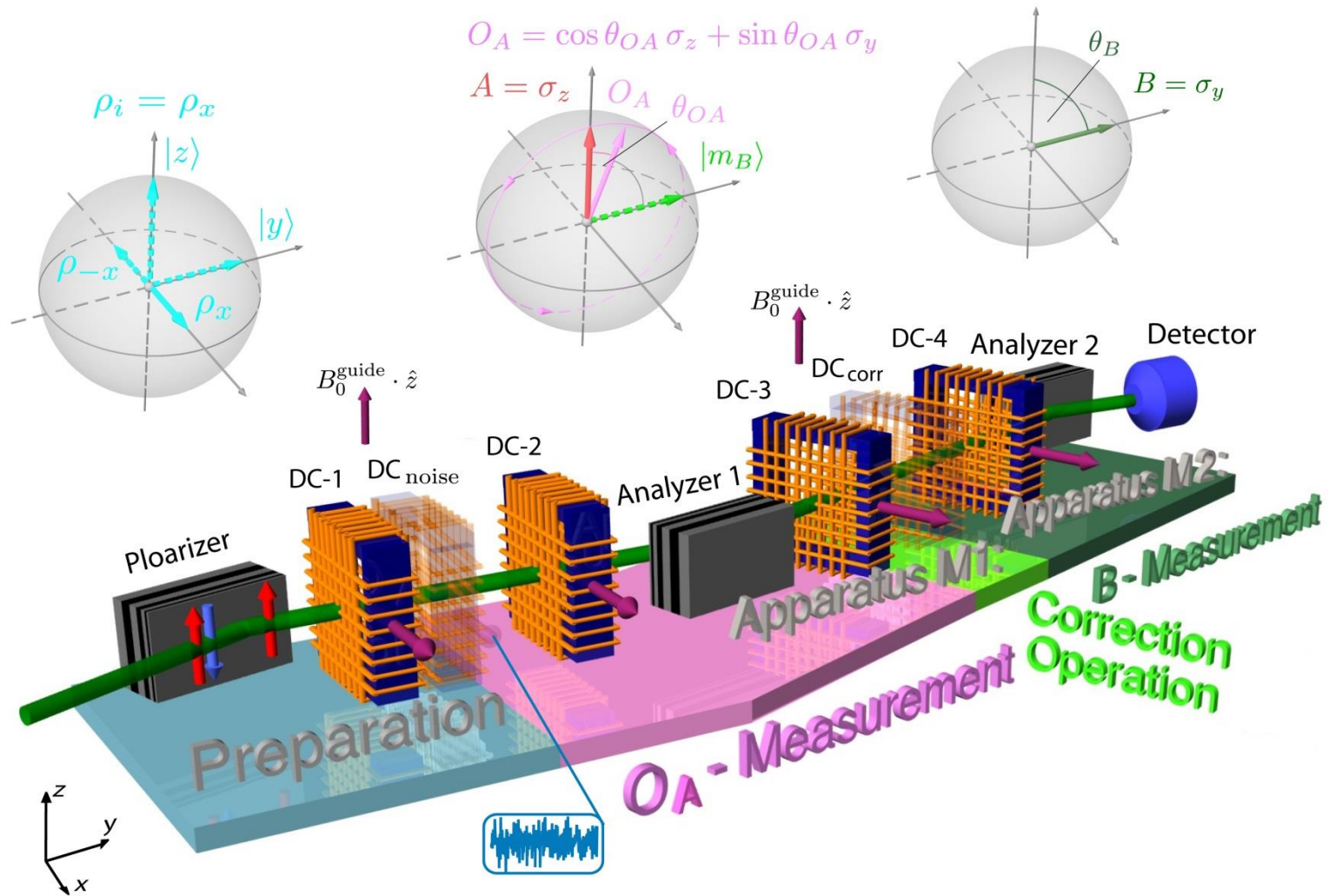
Fumihiko Kaneda,^{1,*} So-Young Baek,^{1,†} Masanao Ozawa,² and Ilya Shchepetov¹

¹Research Institute of Electrical Communication, Tohoku University, Sendai 980-8579, Japan
²Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan
 (Received 27 August 2013; published 15 January 2014)

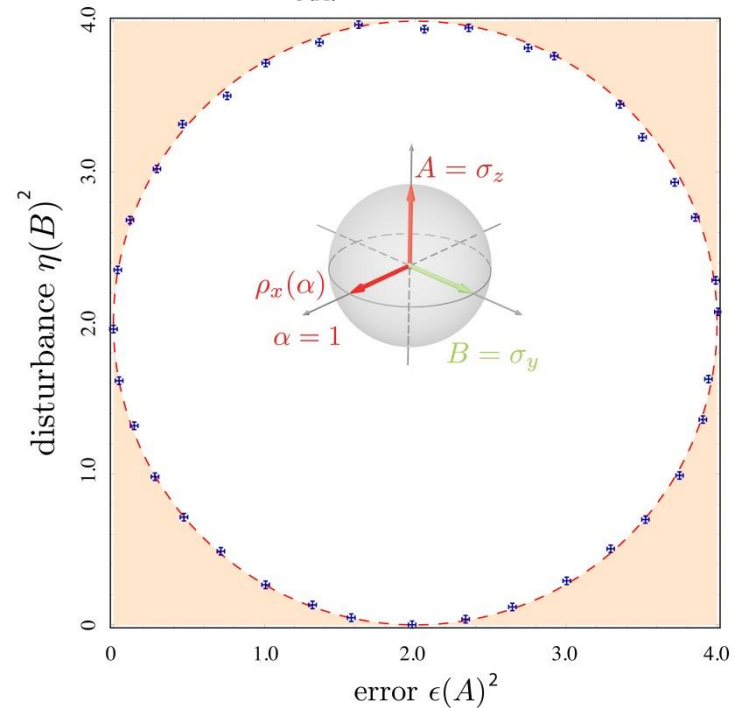
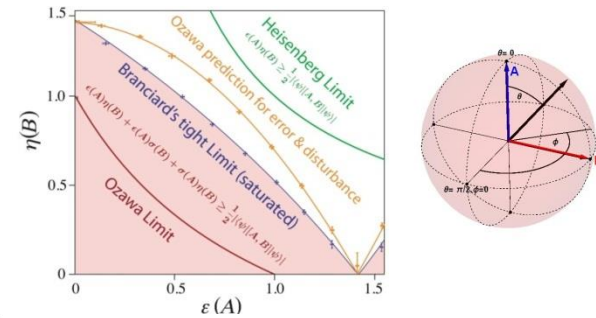
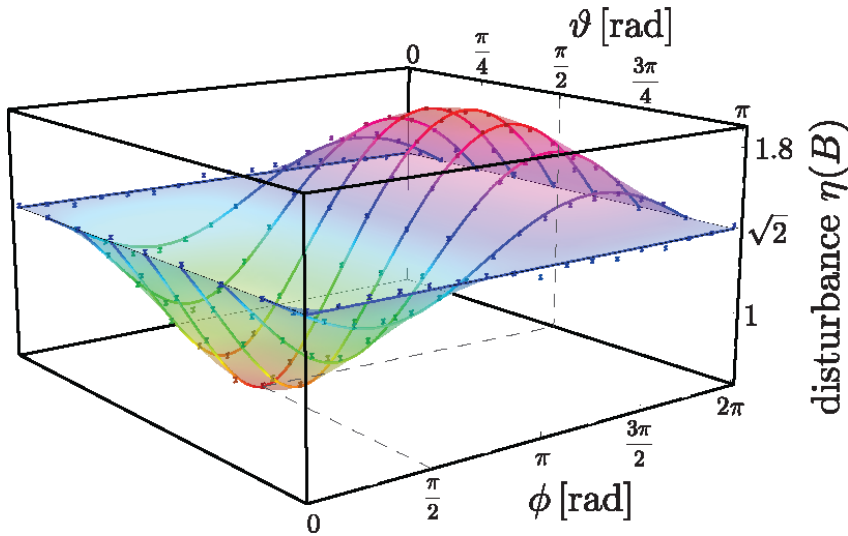
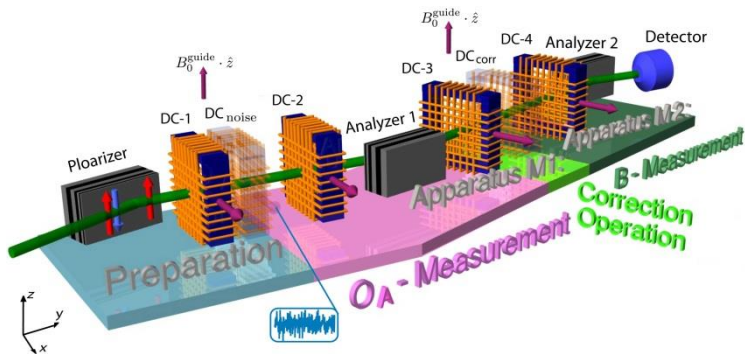
We experimentally test the error-disturbance uncertainty relation (EDR) in the measurement of a single photon polarization qubit, making use of weak measurement. We demonstrate that the Heisenberg EDR is violated, while Branciard EDRs are valid throughout the range of our measurement strength.

C. Branciard, Proc. Natl. Acad. Sci. U.S.A. **110**, 6742 (2013).

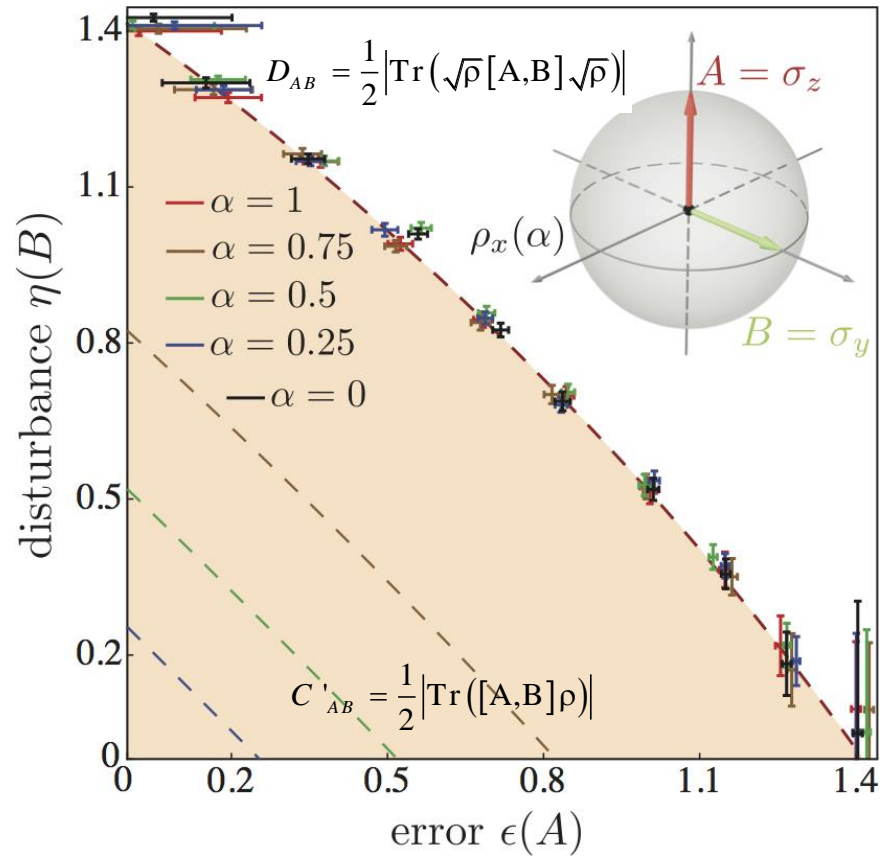
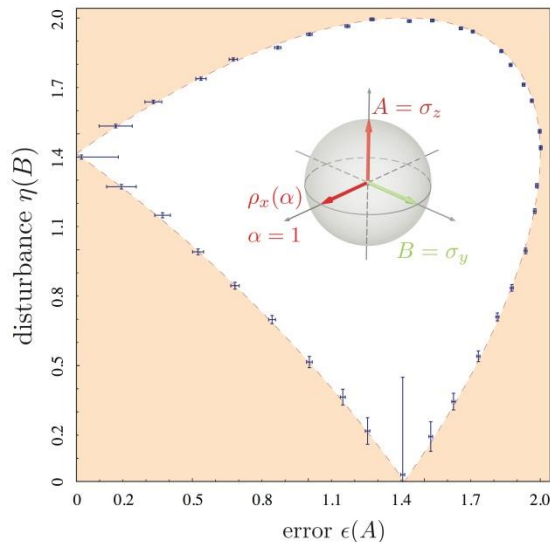
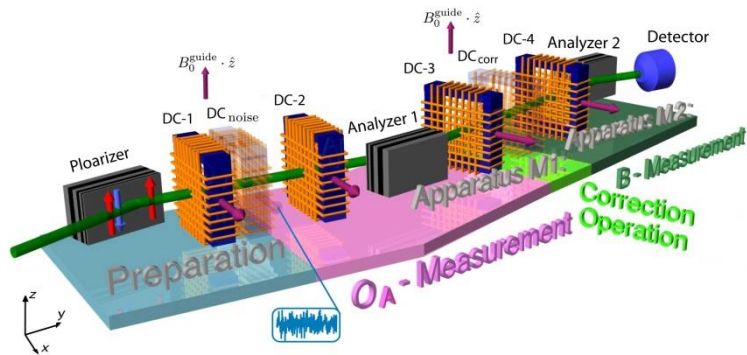
Tight relation: experimental setup



Tight relation: error-corrections

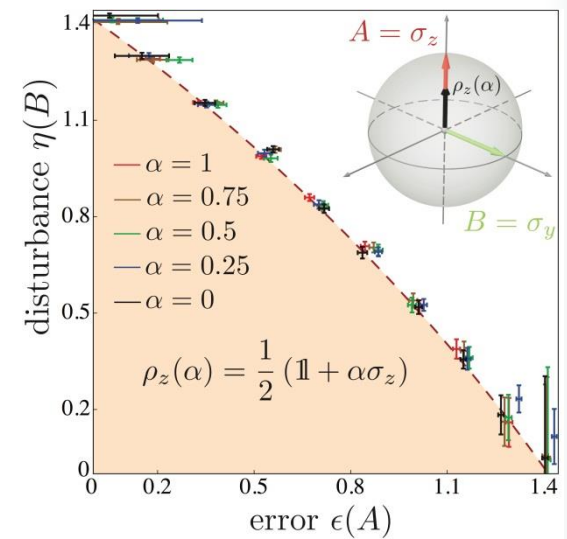
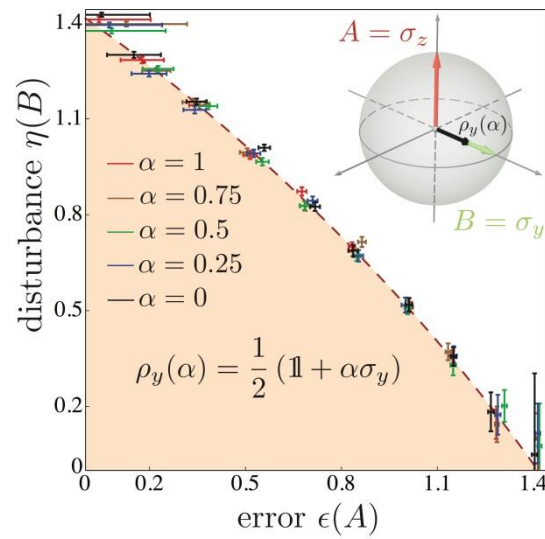
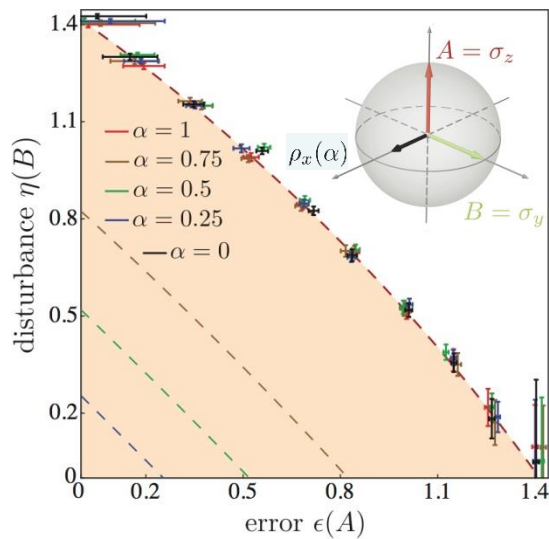
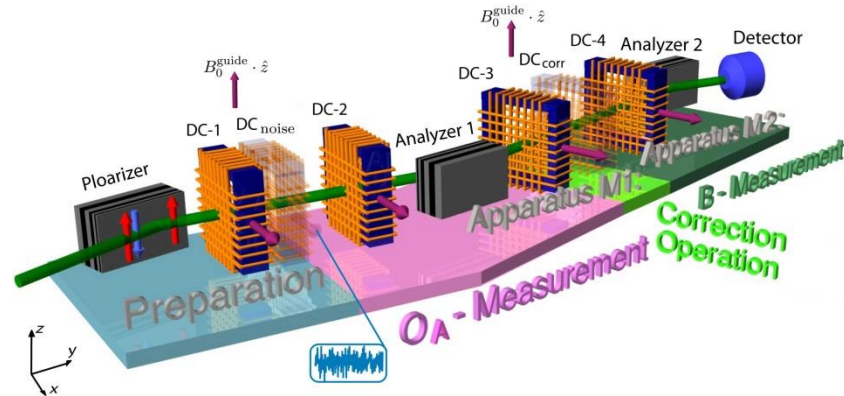


Tight relation: from a pure state to mixed states



Remark: M. Ozawa, arXiv:1404.3388

Tight relation: all mixtures



B. Demirel, PRL 117, 140402 (2016).

Entropic uncertain-relation (UR)

UR for states

❖ Robertson:

$$\Delta A \Delta B \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

❖ Deutsch:

$$H(\mathcal{A}) + H(\mathfrak{B}) \geq -2 \log(c)$$

$$c := \max_{j,k} |\langle a_j | b_k \rangle|$$

UR for measurements

❖ Ozawa: *arXiv: 1404.3388v1*, (2014)

$$\varepsilon(A)^2 \Delta B^2 + \eta(B)^2 \Delta A^2 + 2\varepsilon(A)\eta(B) \sqrt{\Delta A^2 \Delta B^2 - D_{AB}^2} \geq D_{AB}^2$$
$$D_{AB} := \frac{1}{2} \text{Tr}(|\sqrt{\rho}[A, B]\sqrt{\rho}|)$$

❖ Buscemi, Hall: PRL 112, 050401 (2014)

$$N(M, A) + D(M, B) \geq -\log(c)$$

$$N(M, A) := H(\mathcal{A}|\mathcal{M}) \text{ \& \ } D(M, B) := H(\mathfrak{B}|\mathcal{M})$$

Information-theoretic Entropy

Shannon Entropy H :

where $A|a\rangle = a|a\rangle$ for the observable A .

$$H(\mathcal{A}, |\psi\rangle) := -\sum_a p(a) \log(p(a))$$

$$p(a) = |\langle a|\psi\rangle|^2$$

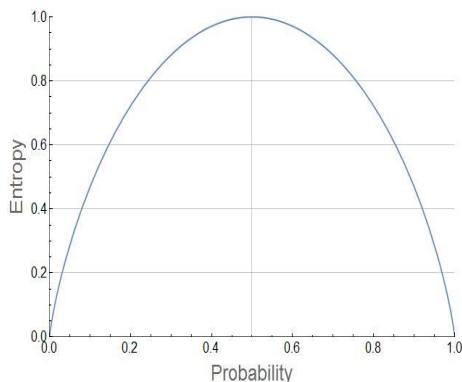
Coin toss: Probability for heads or tails



$$p(\text{heads}) = x \quad p(\text{tails}) = 1-x$$

(Binary) Shannon entropy

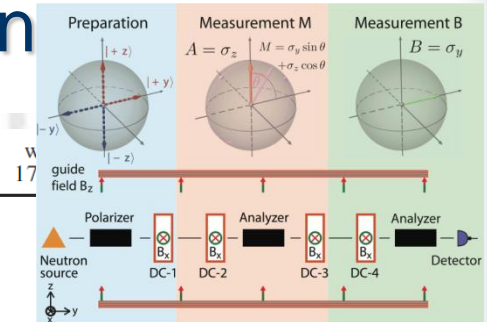
$$H(X) = -x \log(x) - (1-x) \log(1-x)$$



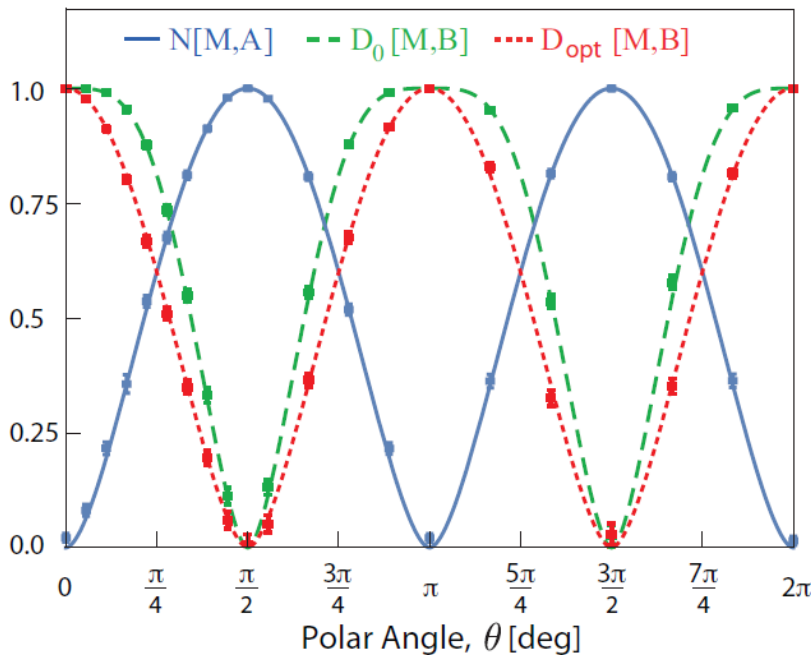
Results for entropic noise-dist. relation

PRL 115, 030401 (2015)

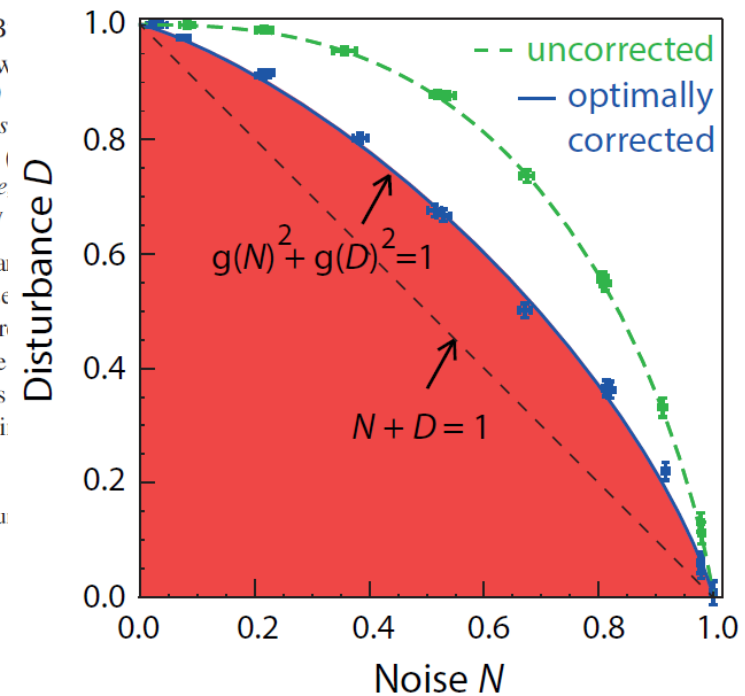
PHYSICAL REVIEW LETTERS



Experimental Test of Entropic Noise-Disturbance Uncertainty Relations for Spin-1/2 Measurements



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Entropic noise-dist. uncertainty relation has π -periodicity !!!

Tight relation is attained.

Improvements with general POVMs

PHYSICAL REVIEW A **94**, 062110 (2016)

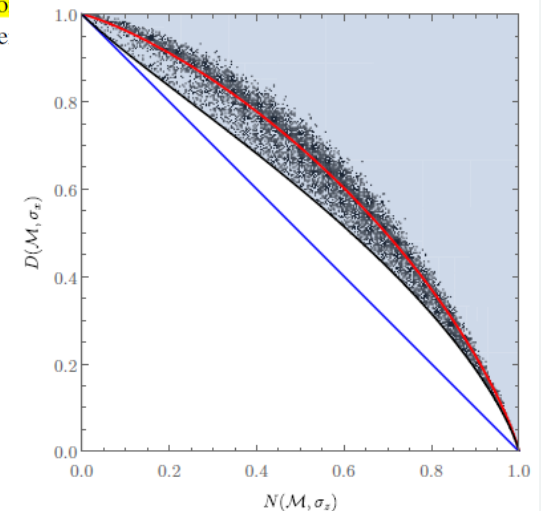
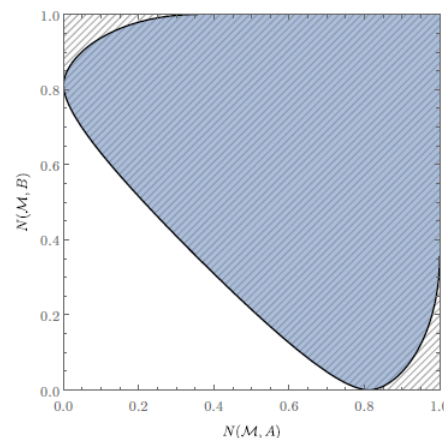
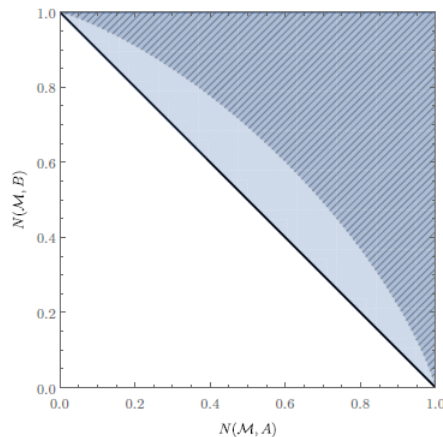
Noise and disturbance of qubit measurements: An information-theoretic characterization

Alastair A. Abbott* and Cyril Branciard

Institut Néel, CNRS and Université Grenoble Alpes, 38042 Grenoble Cedex 9, France

(Received 5 October 2016; published 12 December 2016)

Information-theoretic definitions for the noise associated with a quantum measurement and the corresponding disturbance to the state of the system have recently been introduced [F. Buscemi *et al.*, *Phys. Rev. Lett.* **112**, 050401 (2014)]. These definitions are invariant under relabeling of measurement outcomes, and lend themselves readily to the formulation of state-independent uncertainty relations both for the joint estimate of observables (noise-noise relations) and the noise-disturbance tradeoff. Here we derive such relations for incompatible qubit observables, which we prove to be tight in the case of joint estimates, and present progress towards fully characterizing the noise-disturbance tradeoff. In doing so, we show that the set of obtainable noise-noise values for such observables is convex, whereas the conjectured form for the set of obtainable noise-disturbance values is not. Furthermore, projective measurements are not optimal with respect to the joint-measurement noise or noise-disturbance tradeoffs. Interestingly, it seems that four-outcome measurements are needed in the former case, whereas three-outcome measurements are optimal in the latter.



Results for entropic noise-noise relation: general POVMs

Experimental test of an entropic measurement uncertainty relation for arbitrary qubit observables

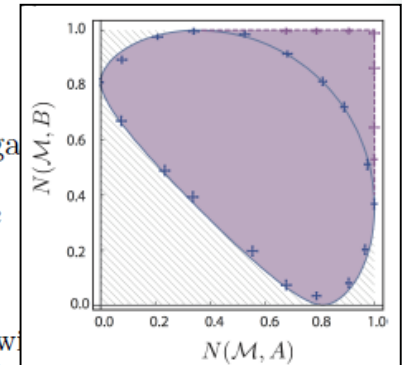
Bülent Demirel¹, Stephan Sponar¹, Alastair A. Abbott², Cyril Branciard², and Yuji Hasega

¹*Atominstytut, TU Wien, Stadionallee 2, 1020 Vienna, Austria*

²*University Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France*

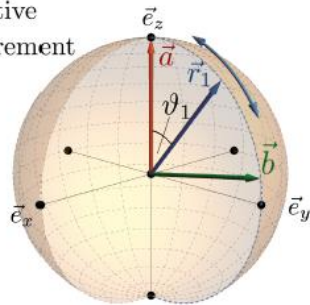
³*Department of Applied Physics, Hokkaido University, Kita-ku, Sapporo 060-8628, Japan*

(Dated: November 15, 2017)



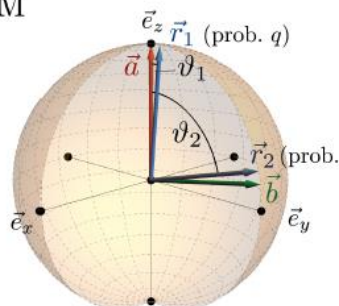
(a)

Projective Measurement

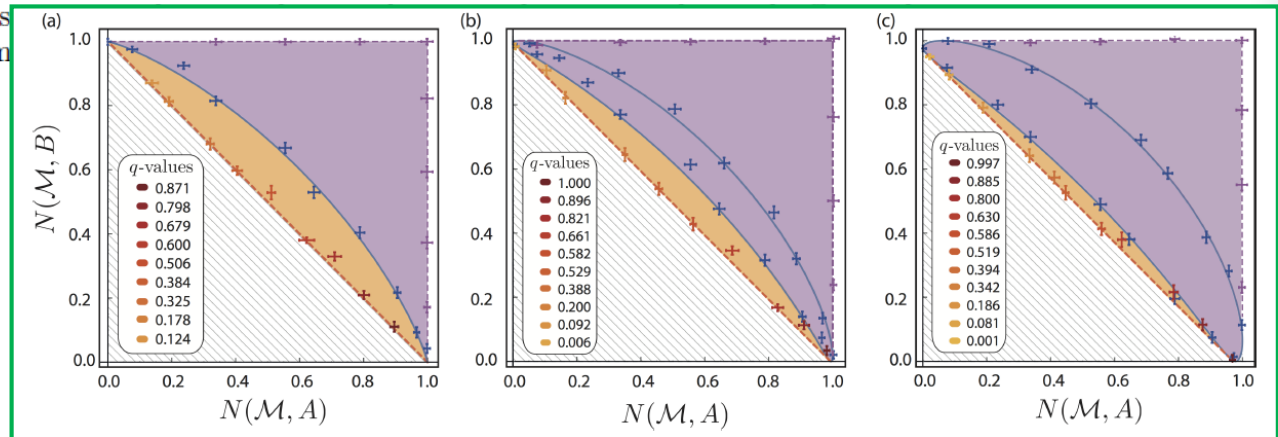


(b)

POVM



entropic measurement uncertainty relation is experimentally tested with the noise associated to the measurement of an observable is defined via $N(M, A)$ and $N(M, B)$ and a tradeoff relation between the noises for two arbitrary spin observables. The optimal bound of this tradeoff is experimentally obtained for arbitrary observables. For some of these observables this lower bound can be



B. Demirel, arXiv:1711.05023

Concluding remarks

Neutron optical experiments are effective methods for studies of foundation of quantum mechanics.

- Quantum dynamics:

quantum Cheshire-cat and pigeonhole effect are observed.

- Error-disturbance uncertainty relation:

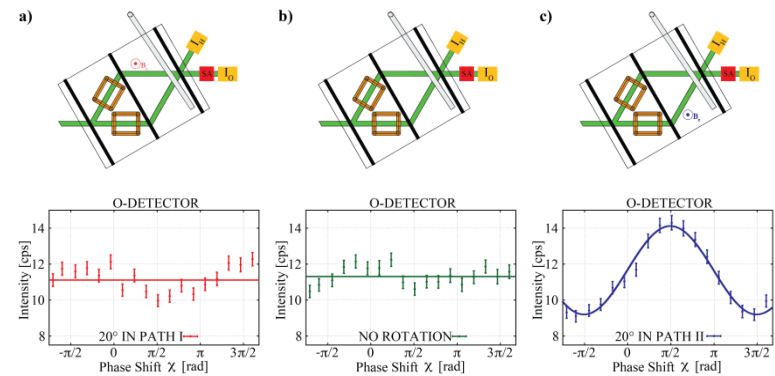
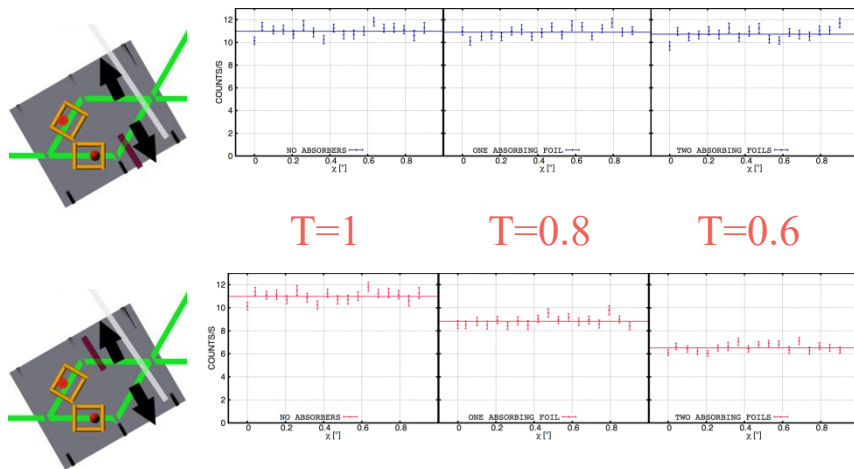
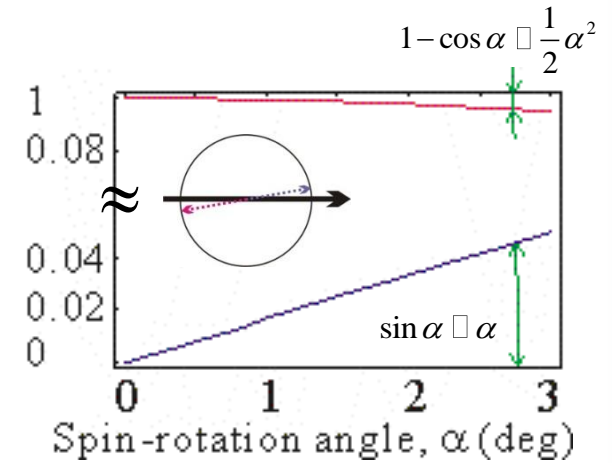
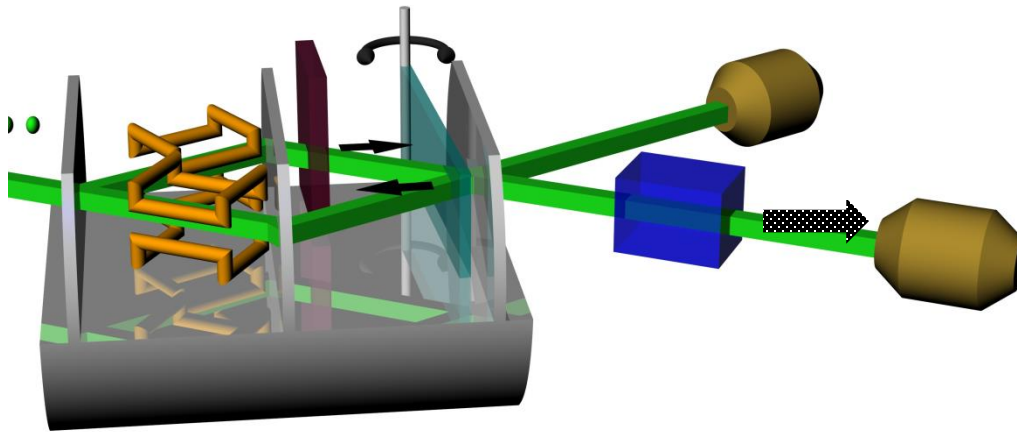
tight relations for pure/mixed states are shown.

- Entropic noise-disturbance uncertainty relation:

tight relations for projective/POVMs are confirmed.

FINA

Another view of quantum Cheshire-cat: effectiveness



Neutron interferometry

Neutrons

$$m = 1.67 \times 10^{-27} \text{ kg}$$

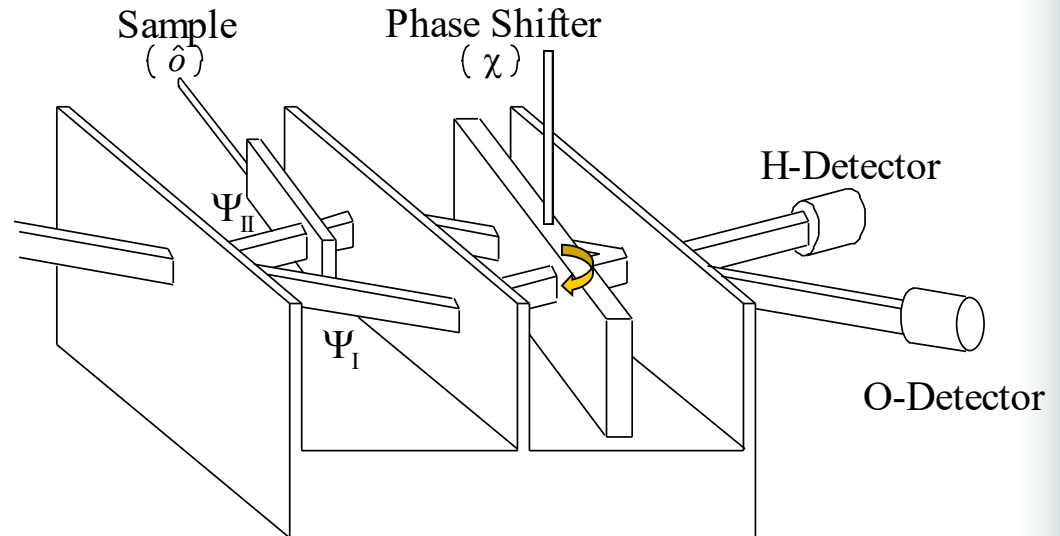
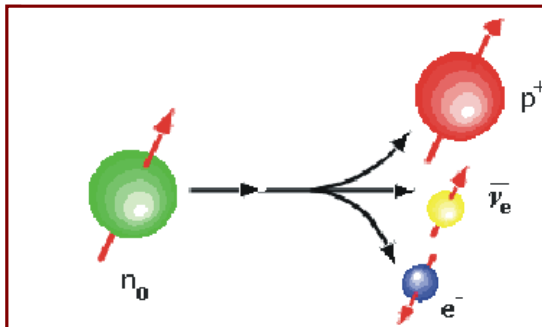
$$s = \frac{1}{2} \hbar$$

$$\mu = -9.66 \times 10^{-27} \text{ J/T}$$

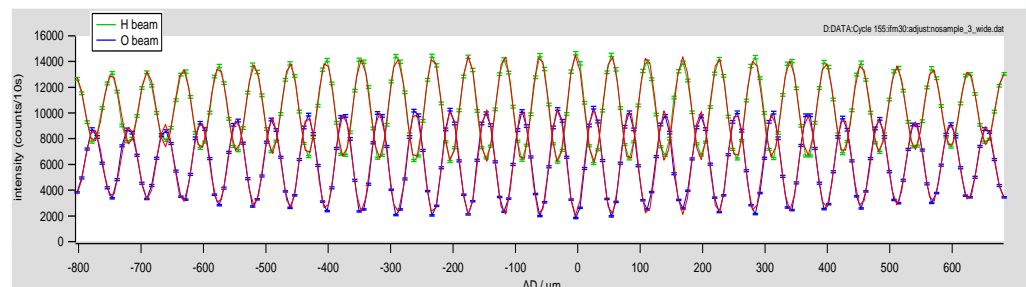
$$\tau = 887 \text{ s}$$

$$R = 0.7 \text{ fm}$$

u-d-d quark structure



$$I = |\Psi_I + e^{i\chi} \cdot \hat{o} \cdot \Psi_{II}|^2$$

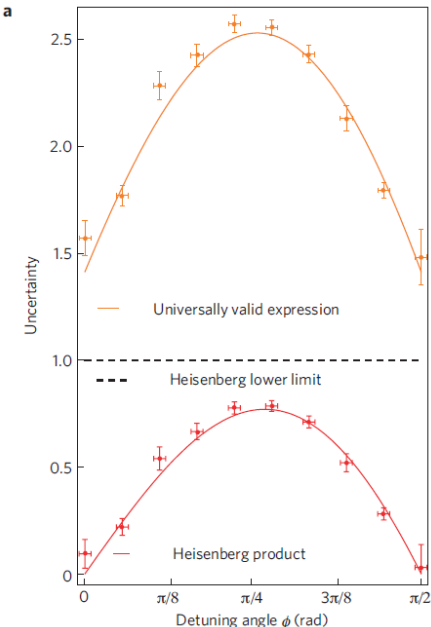
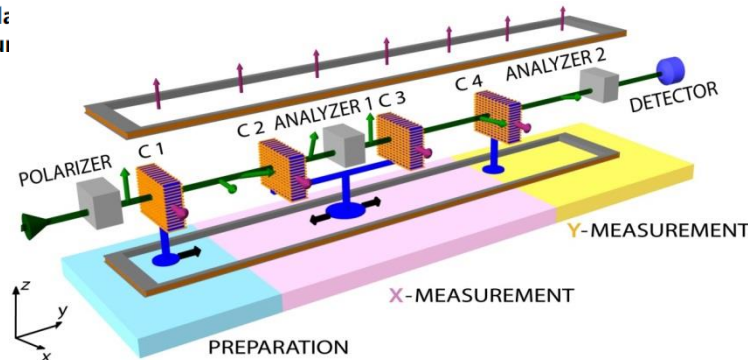


Experimental demonstration of a universally valid error–disturbance uncertainty relation in spin measurements

Jacqueline Erhart¹, Stephan Sponar¹, Georg Sulyok¹, Gerald Badurek¹, Masanao Ozawa² and Yuji Hasegawa^{1*}

The uncertainty principle generally prohibits simultaneous measurements of certain pairs of observables and forms the basis of indeterminacy in quantum mechanics¹. Heisenberg's original formulation, illustrated by the famous γ -ray microscope, sets a lower bound for the product of the measurement error and the disturbance². Later, the uncertainty relation was reformulated in terms of standard deviations^{3–5}, where the focus was exclusively on the indeterminacy of predictions, whereas the unavoidable recoil in measuring devices has been ignored⁶. A correct formulation of the error–disturbance uncertainty relation for a deeper understanding

as $\sigma(A)^2 = \langle \psi | A^2 | \psi \rangle - \langle \psi | A | \psi \rangle^2$. Note that a covariance term can be added to the right-hand side if squared, as discussed by Schrödinger⁵. For setting, this term vanishes. Robertson's relation of standard deviations has been confirmed by measurements. In a single-slit diffraction experiment¹⁵, as expressed in equation (2), has been confirmed and many experimental demonstrations have been reported. Robertson's relation (equation (2)) has a more general application for limitation of simultaneous measurements that is generally understood as



Neutron Quantum Optics generation



Yuji
Hasegawa



Sam Werner



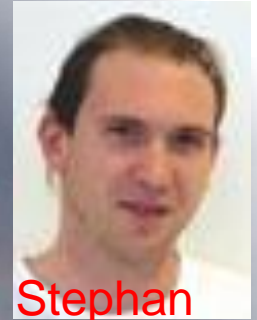
Helmut
Rauch



Gerald
Badurek



Jürgen
Klepp



Stephan
Sponar



Masanao
Ozawa



Michael
Zawisky



Katharina
Durstberger



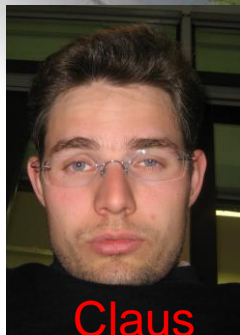
Hartmut
Lemmel



Georg
Sulyok



Daniel
Erdösi



Claus
Schmitzer



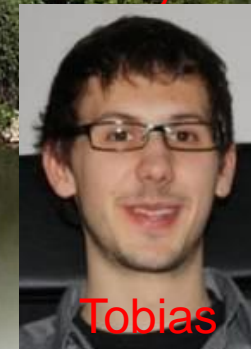
Hannes
Bartosik



Jacqueline
Erhart



Bülent
Demirel



Tobias
Denkmayr



Hermann
Geppert