Int'l Symp. on New Frontiers in Quantum Correlations (ISNFQC18) @S.N. Bose Nat'l Center, Kolkata
29. January 2018

Fundamental Phenomena in Quantum Mechnics studied with Matter-Wave Optical Setup
--- Quantum Cheshire-Cat and Uncertainty Relations ---

Yuji HASEGAWA
Atominsitut, TU-Wien, Vienna, AUSTRIA
Hokkaido University, Sapporo, JAPAN

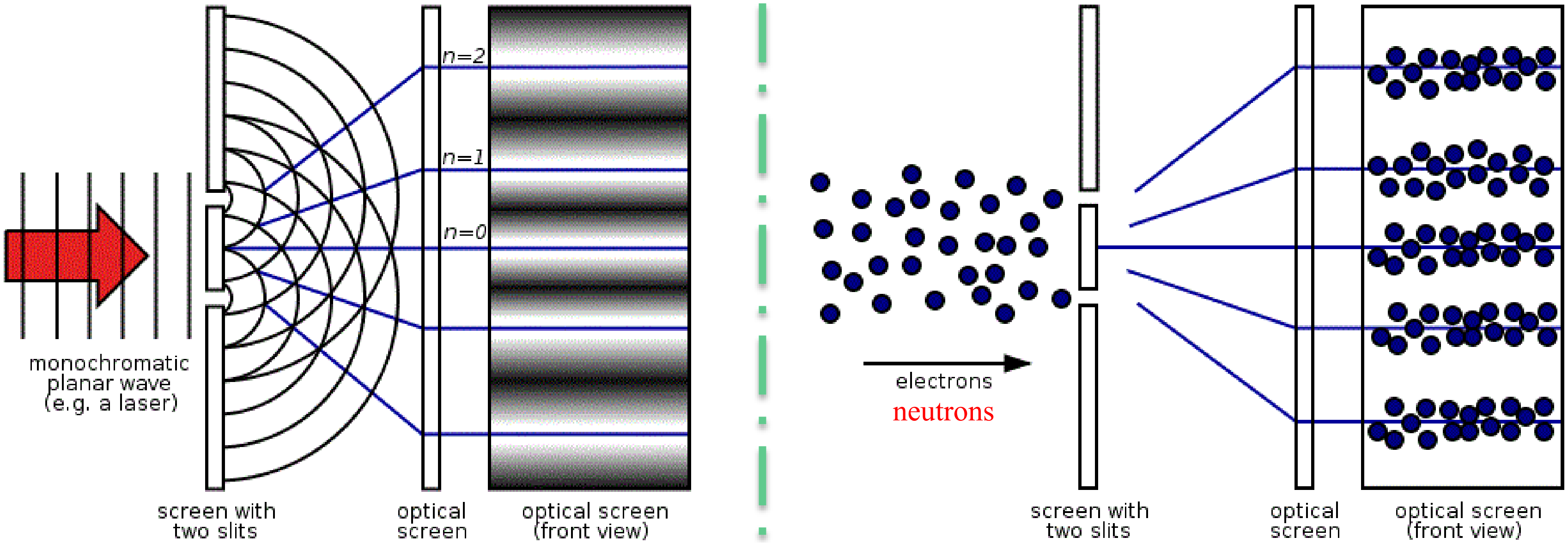
I. **Introduction: neutron as a particle/wave**

II. **Quantum Cheshire-cat & Pigeonhole effect**

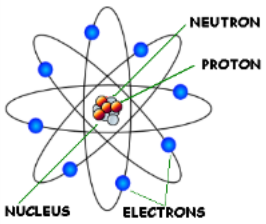
II. **Uncertainty relations for quantum measurements**

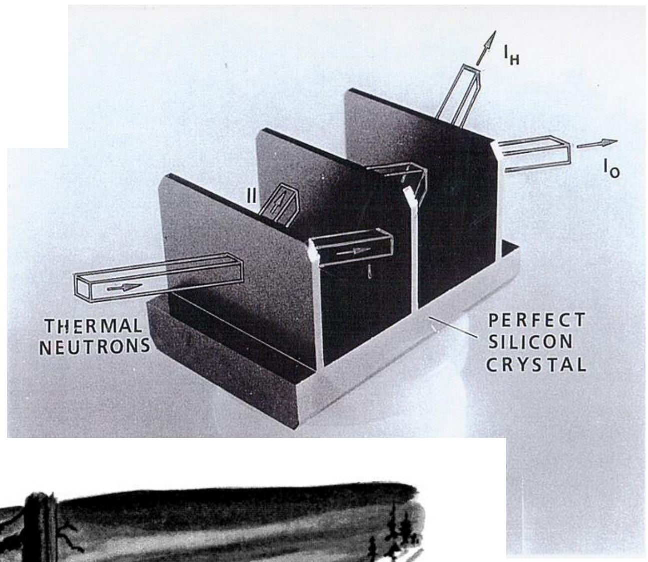
V. **Summary**

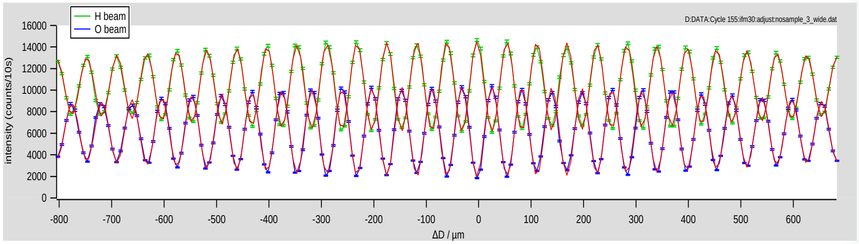
Waves/Nonlocality in classical- and quantum-mechanics

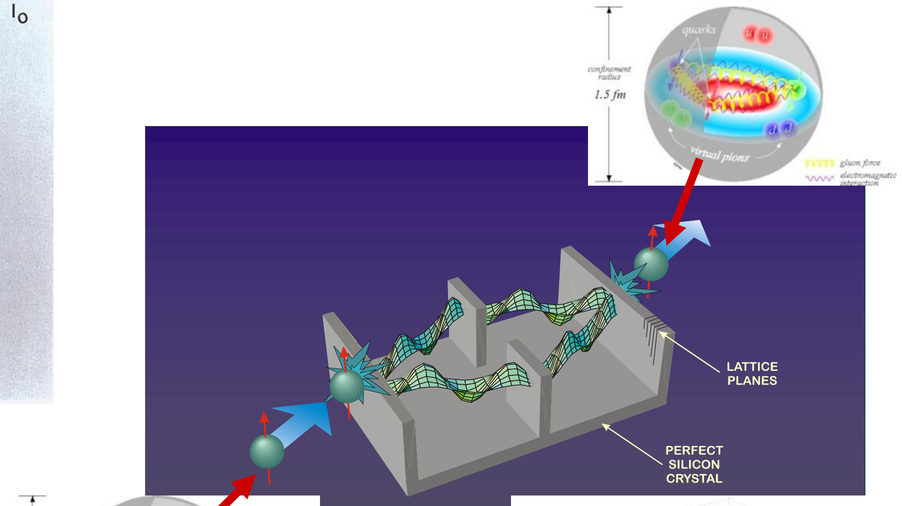


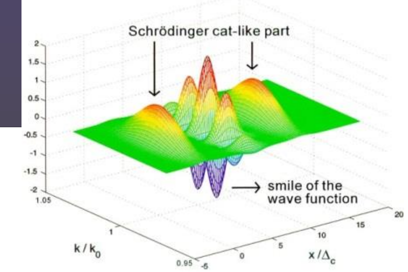
Neutronen interferometry: quantum skier











The New Yorker Collection, Ch.Adams 1940

The neutron

Particle

Feels four-forces

Wave

$m = 1.674928(1) \times 10^{-27}$ kg

$s = \frac{1}{2}\hbar$

$\mu = -9.6491783(18) \times 10^{-27}$ J/T

$\tau = 887(2)$ s

$R = 0.7$ fm

$\alpha = 12.0\,(2.5) \times 10^{-4}$ fm^3

u - d - d - quark structure

CONNECTION

de Broglie

$$\lambda_B = \frac{h}{m.v}$$

Schrödinger

$$H\psi(\vec{r},t) = i\hbar \frac{\delta\psi(\vec{r},t)}{\delta t}$$

&

boundary conditions

$$\lambda_c = \frac{h}{m.c} = 1.319695\,(20) \times 10^{-15}\ \text{m}$$

For thermal neutrons
$\lambda = 2$Å, v= 2km/s, $E_{kin} = 20$meV

$$\lambda_B = \frac{h}{m.v} = 1.8 \times 10^{-10}\ \text{m}$$

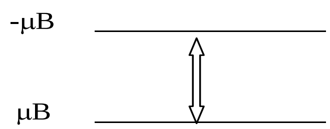
$$\Delta_c = \frac{1}{2\delta k} \cong 10^{-8}\ \text{m}$$

$$\Delta_p = v.\Delta t \cong 10^{-2}\ \text{m}$$

$$\Delta_d = v.\tau = 1.942(5) \times 10^{6}\ \text{m}$$

$$0 \le \chi \le 2\pi\ (4\pi)$$

m ... mass, s ... spin, μ ... magnetic moment, τ ... β-decay lifetime, R ... (magnetic) confinement radius, α ... electric polarizability; all other measured quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero



two level system

λ_c ... Compton wavelength, λ_B ... deBroglie wavelength, Δ_c ... coherence length, Δ_p ... packet length, Δ_d ... decay length, δk.... momentum width, Δt ... chopper opening time, v ... group velocity, χ ......phase.

Neutrons in quantum mechanics

Particle and wave properties

$$p = mv = h/\lambda$$

(L. De Broglie)

Schroedinger equation

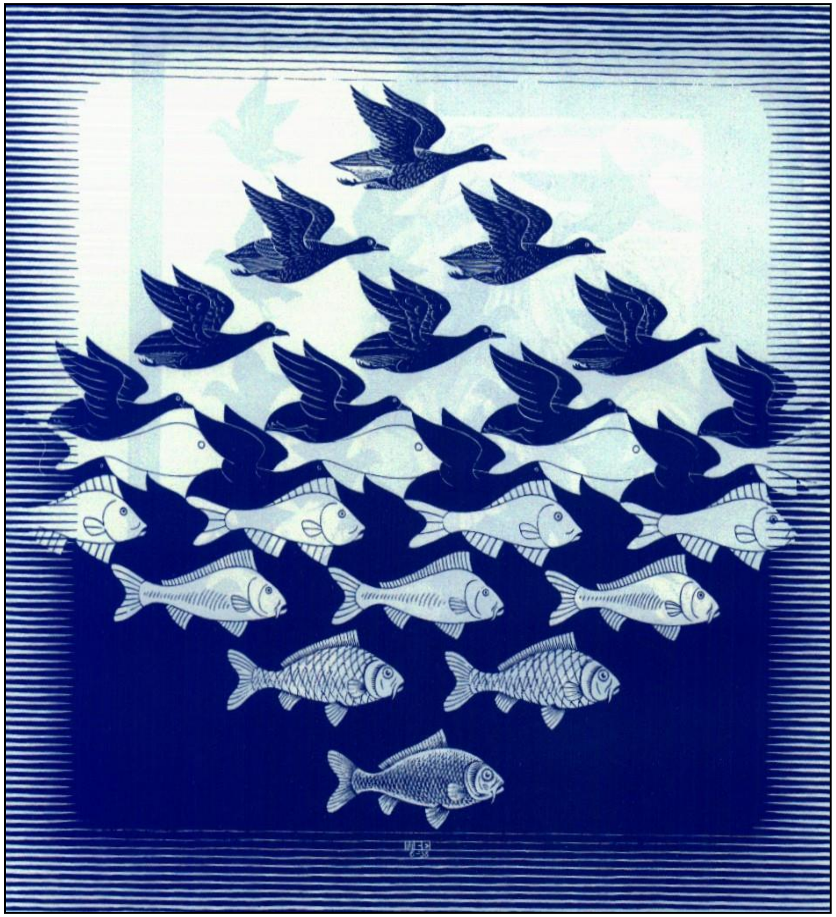
$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = H\Psi(\vec{r},t)$$

(E. Schrödinger)

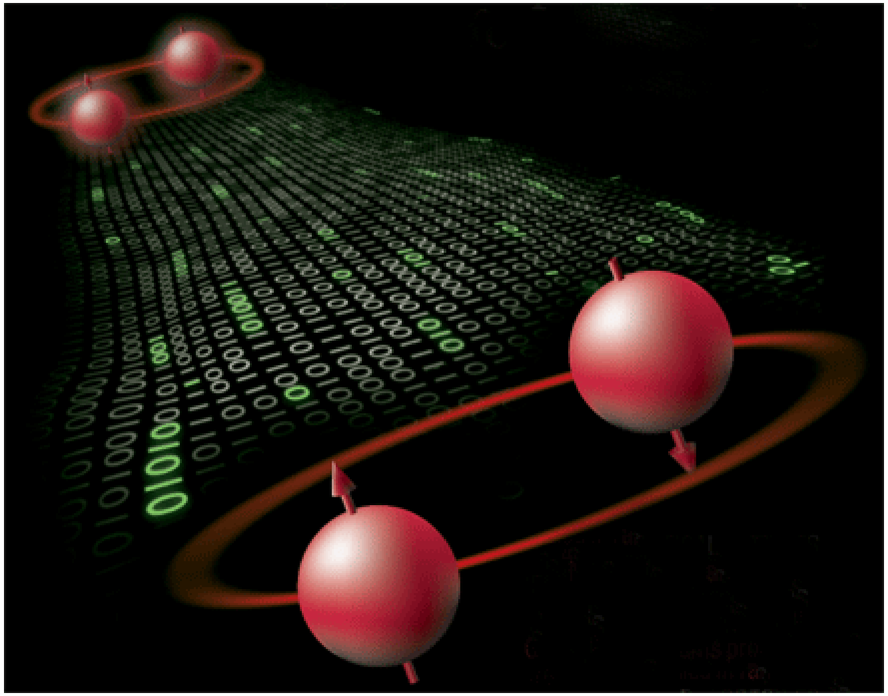
Uncertainity

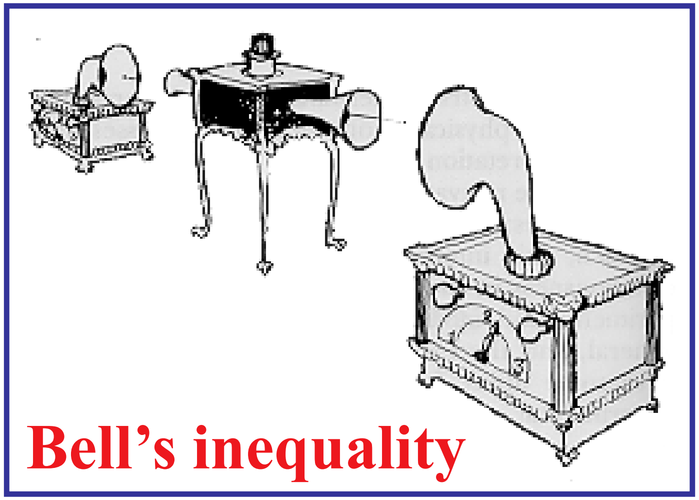
$$\Delta x \, \Delta p \geq \hbar/2$$

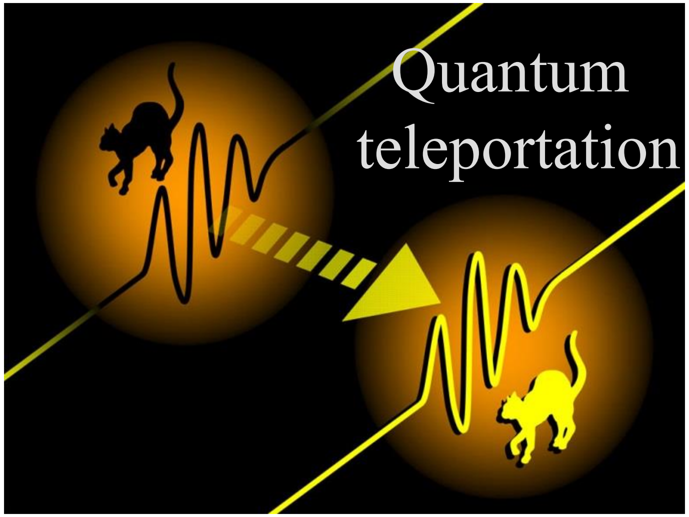
(W. Heisenberg)



Quantum information technology: entanglement







Bi-partite and tri-partite entanglements

2-Particle Bell-State

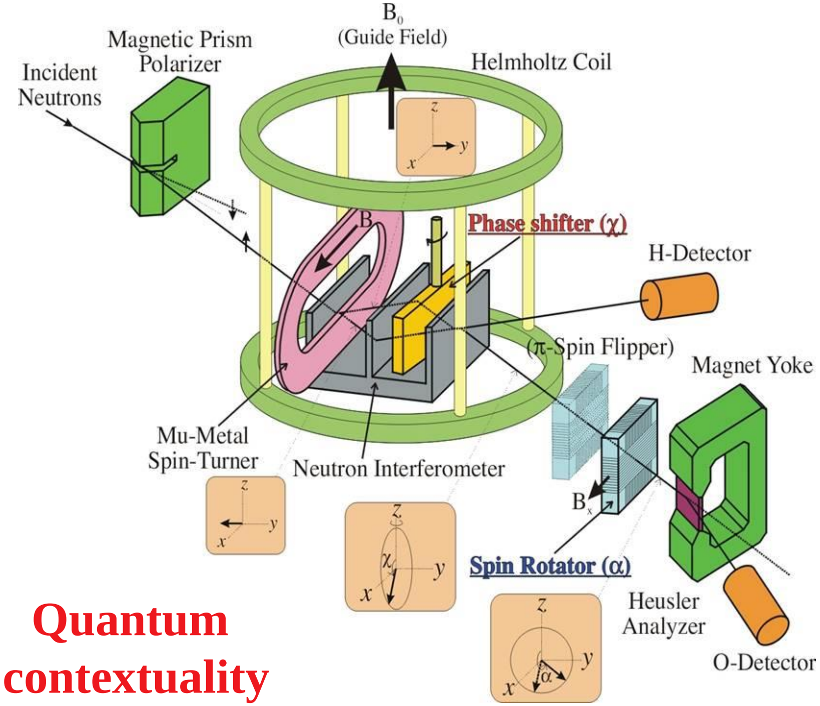
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{I}\otimes|\downarrow\rangle_{II} + |\downarrow\rangle_{I}\otimes|\uparrow\rangle_{II}\}$$

I, II represent 2-Particles

2-Space Bell-State

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{s}\otimes|\,\mathrm{I}\rangle_{p} + |\downarrow\rangle_{s}\otimes|\,\mathrm{II}\rangle_{p}\}$$

s, p represent 2-Spaces, e.g., spin & path



Quantum contextuality

Violation of Bell-like inequality

$$S' \equiv E'(\alpha_1,\chi_1) + E'(\alpha_1,\chi_2) - E'(\alpha_2,\chi_1) + E'(\alpha_2,\chi_2)$$
$$= \mathbf{2.051 \pm 0.019 > 2}$$

Nature2003, NJP2011

Kochen-Specker-like contradiction

$$E_x \cdot E_y = 0.407 \xleftrightarrow{63\%} E' \equiv \langle \hat{X}_1\hat{Y}_2 \cdot \hat{Y}_1\hat{X}_2 \rangle = -0.861$$

PRL2006/2009

Tri-partite entanglement (GHZ-state)

$$\left|\Psi_{\mathrm{Neutron}}\right\rangle = \{|\Psi_{I}\rangle\otimes|\uparrow\rangle\otimes|\Psi(E_0)\rangle + \left(e^{i\chi}|\Psi_{II}\rangle\right)\otimes\left(e^{i\alpha}|\downarrow\rangle\right)\otimes\left(e^{i\gamma}|\Psi(E_0+\hbar\omega_r)\rangle\right)\}$$

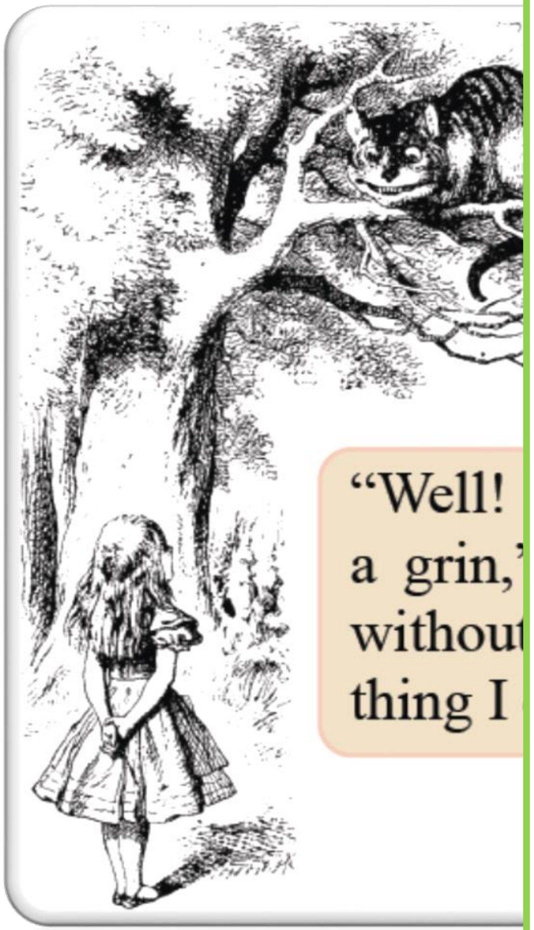
$$M_{Measured} = 2.558 \pm 0.004 > 2$$

PRA2010/NJP2013

Cheshire-cat

"Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in all my life!"

Quantum Cheshire-cat



New Journal of Physics
The open access journal for physics

Quantum Cheshire Cats

Yakir Aharonov[1,2], Sandu Popescu[3,6], Daniel Rohrlich[4,6] and Paul Skrzypczyk[5]

[1] Tel Aviv University, School of Physics and Astronomy, Tel Aviv 69978, Israel
[2] Schmid College of Science, Chapman University, 1 University Drive, Orange, CA 92866, USA
[3] H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom
[4] Physics Department, Ben Gurion University of the Negev, Beersheba, Israel
[5] Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
E-mail: s.popescu@gmail.com and rohrlich@bgu.ac.il

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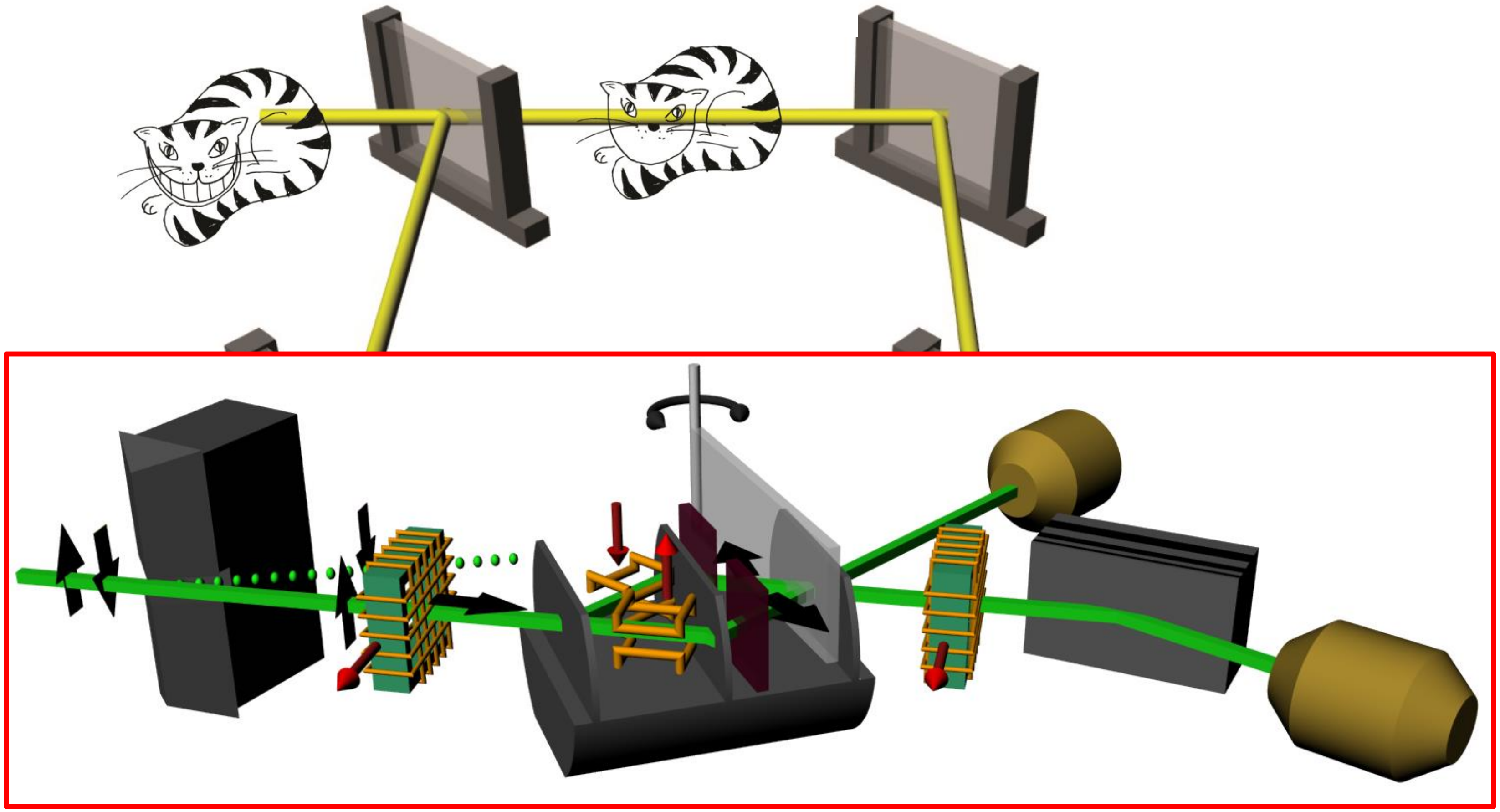
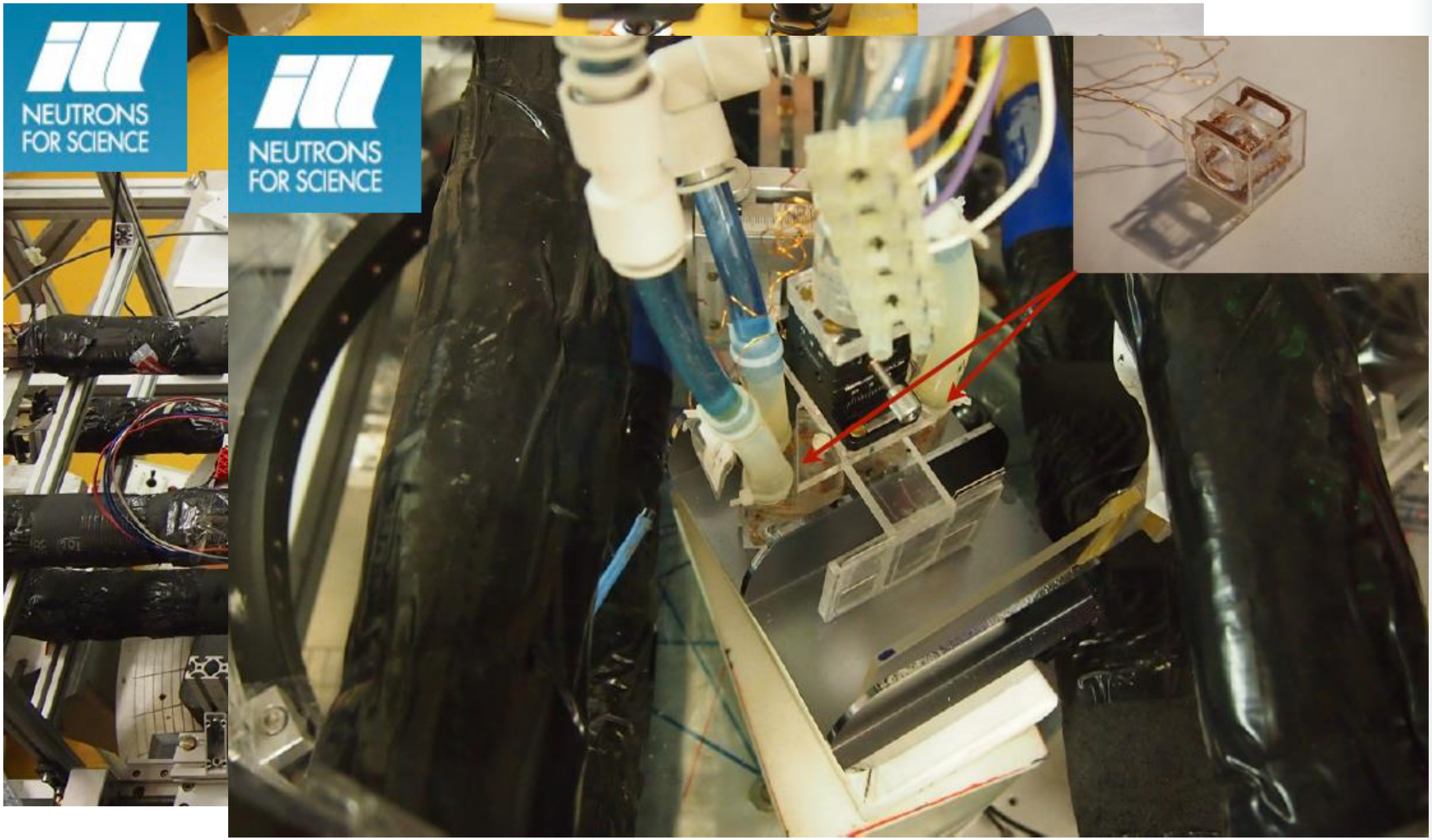
Abstract. In this paper we present a quantum Cheshire Cat. In a pre- and post-selected experiment we find the Cat in one place, and its grin in another. The Cat is a photon, while the grin is its circular polarization.

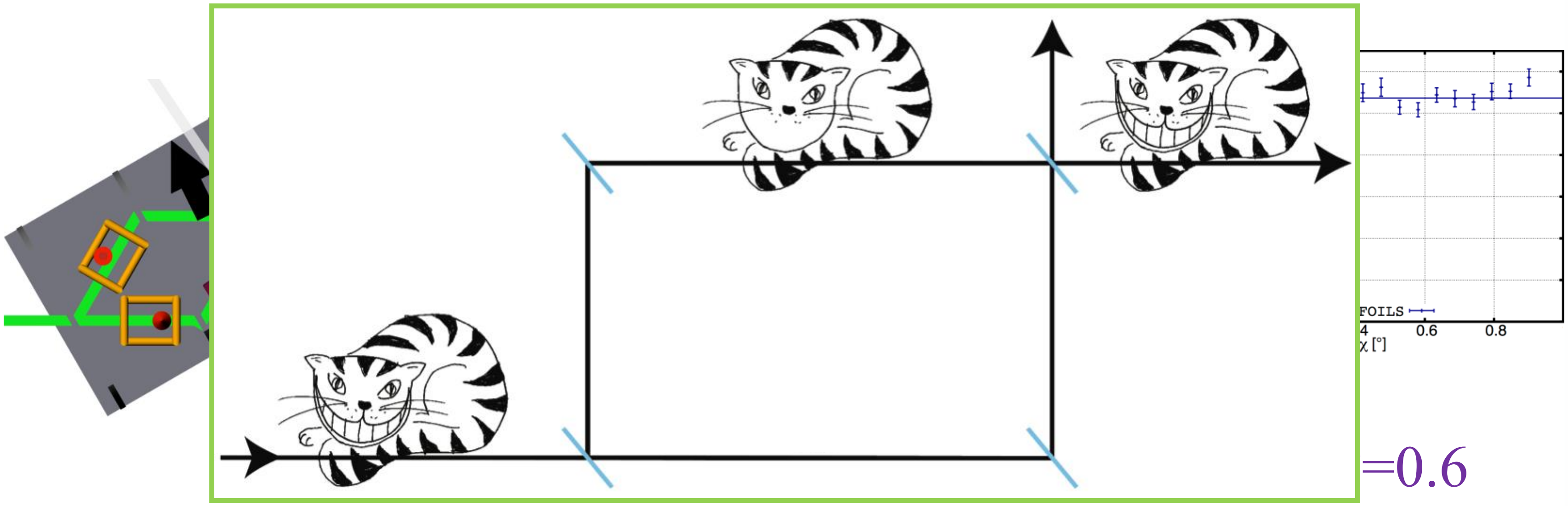
Quantum Cheshire-cat in neutron interferometer



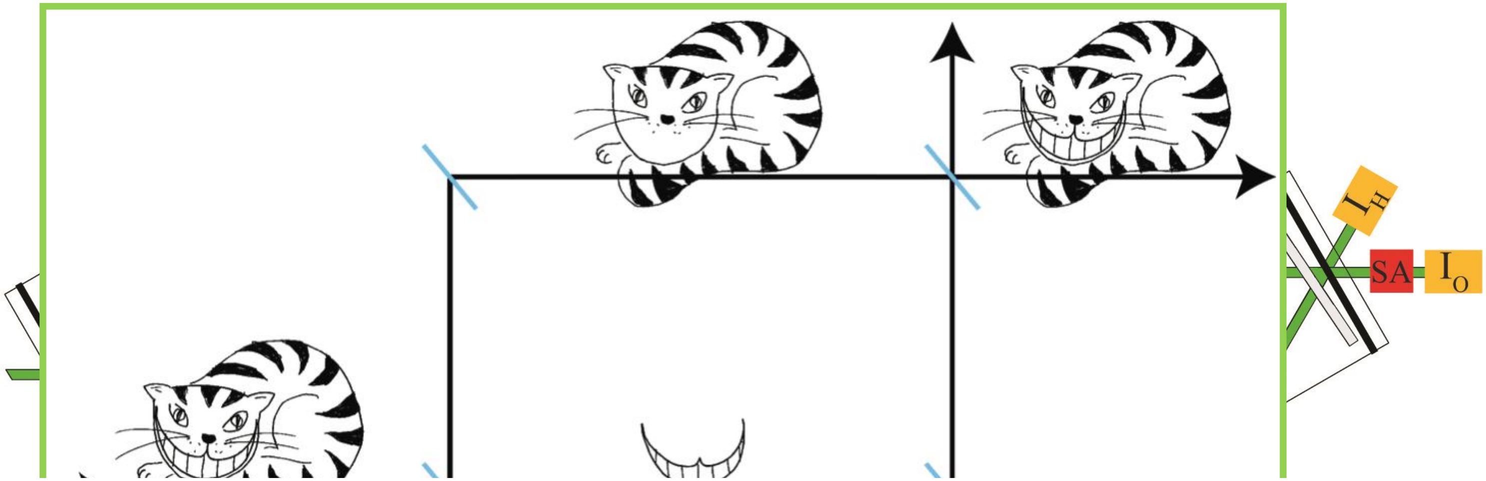
Quantum Cheshire-cat: experiment



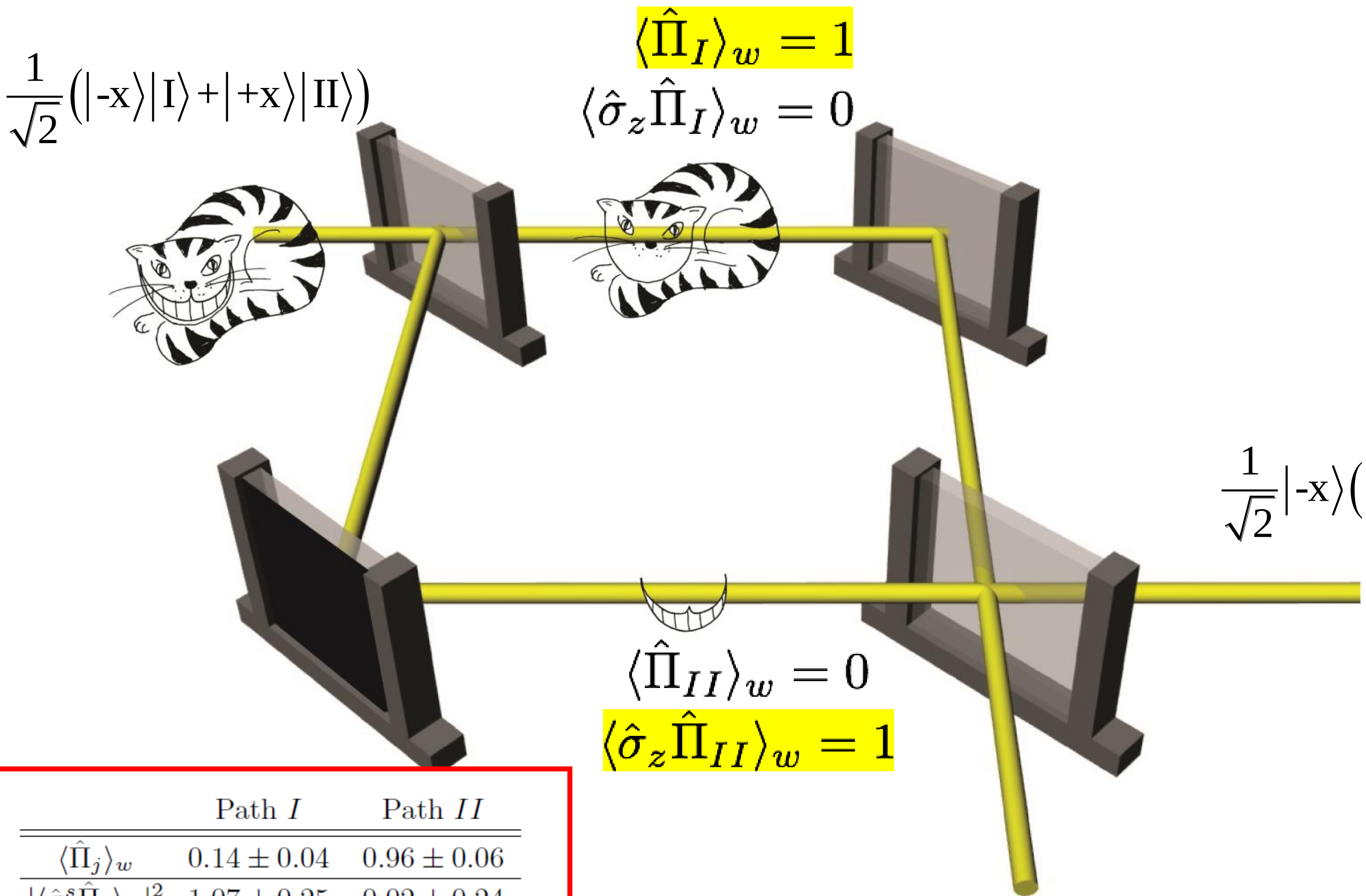
Quantum Cheshire-cat: neutron(cat) in upper path



Quantum Cheshire-cat: spin(smile) in lower path



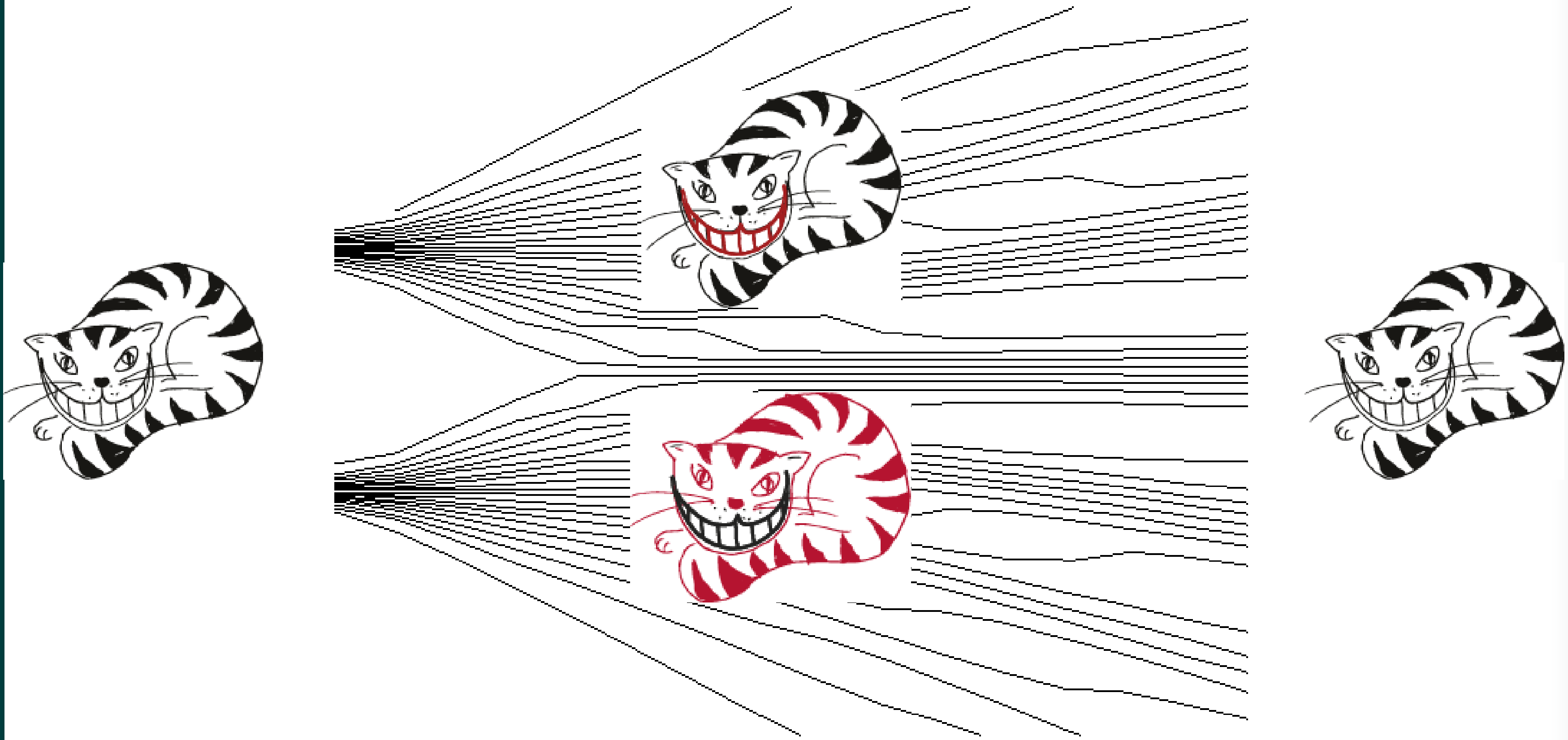
Quantum Cheshire-cat: final results



	Path I	Path II
$\langle\hat{\Pi}_j\rangle_w$	0.14 ± 0.04	0.96 ± 0.06
$\lvert\langle\hat{\sigma}_z^s\hat{\Pi}_j\rangle_w\rvert^2$	1.07 ± 0.25	0.02 ± 0.24

T. Denkmayr et al. Nature Comm. 5:4492 (2014).

Quantum Cheshire-cat: invisible cat/spin ???



http://Bohmian-mechanics.net/

Reactions


BBC NEWS
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29 July 2014 Last upd
'Quantun
By James Morgan
Science reporter, BBC
The Cheshire Cat myste

DER STANDARD FORSCHUNG SPEZIAL MITTWOCH, 30. JULI 2014
Die Katze verschwindet, ihr Grinsen bleibt
Wiener Experiment
Wien – Die Quantenmech
den Menschen schon e

Die Presse SAMSTAG, 16. AUGUST 2014 WISSEN
Wie Krankheiten den Lauf der Geschichte ändern Seite 54
Versteckte Emissionen eines lange verbotenen Treibhausgases Seite 55
Eine übereifrige Qualitätskontrolle, die zu Muskelschwund führt Seite 55
Ein alter Stern war Zeuge einer heftigen Explosion Seite 55

Von grinsenden Katzen und nackten Neutronen
Mithilfe schwacher Messungen schaffen Forscher im Labor das Analogon einer Cheshire Cat

Jeder Physikstudent lernt, dass es unmöglich ist, Informationen über ein Quantenobjekt zu gewinnen, ohne es zu stören. Doch mit schwachen Messungen wird das Unmögliche möglich. Bei der Interpretation der Resultate ist allerdings Vorsicht angebracht.

Christian Speicher

Die Cheshire-Katze und ihre quantenmechanische Entsprechung

Getrennte Wege

Lust auf Salz
Schadet ein geringer Salzkonsum?

Die Frage, wie viel Salz man maximal verzehren sollte, erhitzt seit langem die Gemüter. Öl ins Feuer giessen nun die Initiatoren einer weltumspannenden Bevölkerungsstudie.

Nicola von Lutterotti

Unerwartetes Ergebnis

...stier der Quantenphysiker. [NG Collection/Interfoto/picturedesk.com]

...-Wunderland
...von ihren Eigenschaften zu trennen.
...m Wunderland".

Quantum pigeonhole effect 1

PNAS

Quantum violation of the pigeonhole principle and the nature of quantum correlations

Yakir Aharonov[a,b,c,1], Fabrizio Colombo[d], Sandu Popescu[c,e], Irene Sabadini[d], Daniele C. Struppa[b,c], and Jeff Tollaksen[b,c]

[a]School of … Orange, CA 92866; [c]Ins… …ilan, Italy; and [e]H. H.…

Contribute…

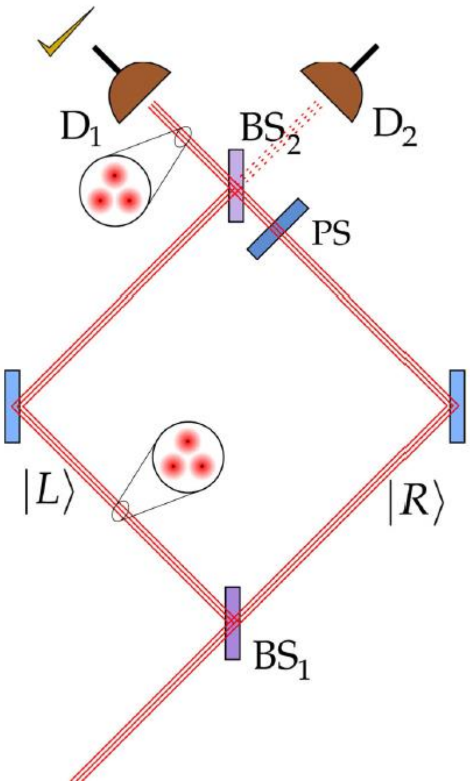
532–535 … s.1522411112

The pigeonhole principle: “If you put three pigeons in two pigeonholes, at least two of the pigeons end up in the same hole,” is an obvious yet fundamental principle of nature as it captures the very essence of counting. Here however we show that in quantum mechanics this is not true! We find instances when three quantum particles are put in two boxes, yet no two particles are in the same box. Furthermore, we show that the above “quantum pigeonhole principle” is only one of a host of related quantum effects, and points to a very interesting structure of quantum mechanics that was hitherto unnoticed. Our results shed new light on the very notions of separability and correlations in quantum mechanics and on the nature of interactions. It also presents a new role for entanglement, complementary to the usual one. Finally, interferometric experiments that illustrate our effects are proposed.

FWF

Quantum pigeonhole effect 2

$$|\Phi\rangle = |+i\rangle_1 |+i\rangle_2 |+i\rangle_3 \quad where \ |\pm i\rangle \equiv \frac{1}{\sqrt{2}}\{|L\rangle \pm i|R\rangle\}$$



$$\begin{cases} \hat{\Pi}_{12}^{same} = \hat{\Pi}_{12}^{LL} + \hat{\Pi}_{12}^{RR} \\ \hat{\Pi}_{12}^{diff} = \hat{\Pi}_{12}^{LR} + \hat{\Pi}_{12}^{RL} \end{cases} \quad where \ \hat{\Pi}_{12}^{LL} \equiv |L\rangle\langle L| \ etc.$$

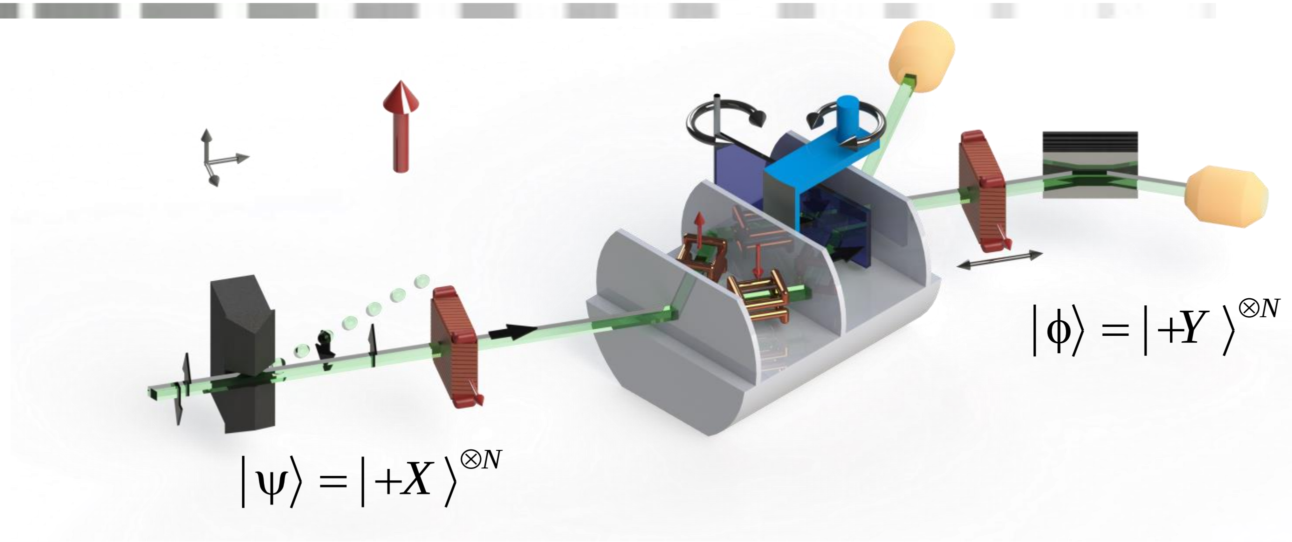
$$\begin{aligned} &\langle\Phi|\hat{\Pi}_{12}^{same}|\Psi\rangle \\ &\propto \left({}_1\langle L| - i\,{}_1\langle R|\right)\left({}_2\langle L| - i\,{}_2\langle R|\right){}_3\langle +| \\ &\quad \times \left\{\left(|L\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2\right)|+\rangle_3\right\} \\ &= \left({}_1\langle L|{}_2\langle L| - i\,{}_1\langle L|{}_2\langle R| - i\,{}_1\langle R|{}_2\langle L| - {}_1\langle R|{}_2\langle R|\right) \\ &\quad \times \left(|L\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2\right) \\ &= 0 \end{aligned}$$

No particles are in the same path!

$$|\Psi\rangle = |+\rangle_1 |+\rangle_2 |+\rangle_3 \quad where \ |\pm\rangle \equiv \frac{1}{\sqrt{2}}\{|L\rangle \pm |R\rangle\}$$

Y. Aharonov et al. PNAS **113**, 532 (2016).

Quantum pigeonhole effect in neutron interferometer



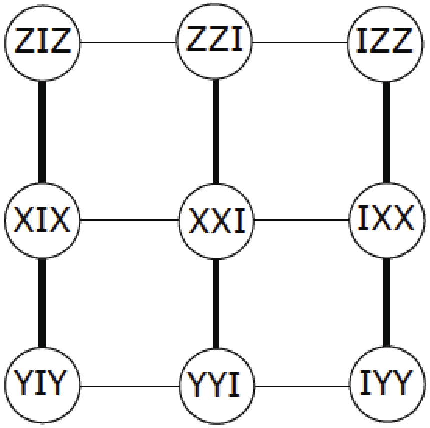
$$\begin{cases} \hat{\Pi}^{even} = \hat{\Pi}_+\hat{\Pi}_+ + \hat{\Pi}_-\hat{\Pi}_- \\ \hat{\Pi}^{odd} = \hat{\Pi}_+\hat{\Pi}_- + \hat{\Pi}_-\hat{\Pi}_+ \end{cases} \quad where \ \hat{\Pi}_\pm \equiv |\pm Z\rangle\langle\pm Z|$$

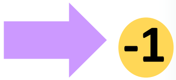
$$\hat{\Pi}_w^{even} = 0 \ \& \ \hat{\Pi}_w^{odd} = 1, \ then \ \left(\hat{\sigma}_z^{path} \bullet \hat{\sigma}_z^{path}\right)_w = -1$$

No two pigeons ever seem to be in the same box!!!

Quantum contextuality in neutron interferometer

(a) Magic square

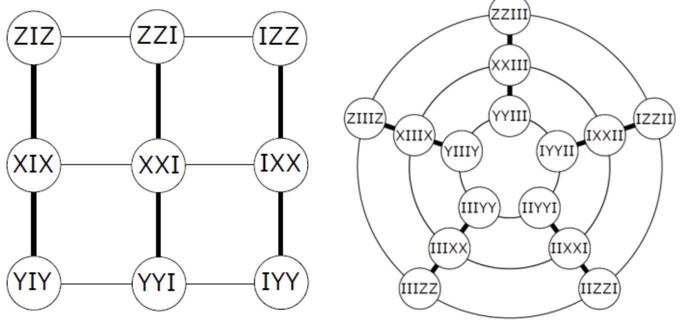




$$\Pi_1^{(3)} = |+Z,+Z,+Z\rangle\langle+Z,+Z,+Z| + |-Z,-Z,-Z\rangle\langle-Z,-Z,-Z|,$$
$$\Pi_2^{(3)} = |+Z,-Z,+Z\rangle\langle+Z,-Z,+Z| + |-Z,+Z,-Z\rangle\langle-Z,+Z,-Z|,$$
$$\Pi_3^{(3)} = |-Z,+Z,+Z\rangle\langle-Z,+Z,+Z| + |+Z,-Z,-Z\rangle\langle+Z,-Z,-Z|,$$
$$\Pi_4^{(3)} = |-Z,-Z,+Z\rangle\langle-Z,-Z,+Z| + |+Z,+Z,-Z\rangle\langle+Z,+Z,-Z|,$$

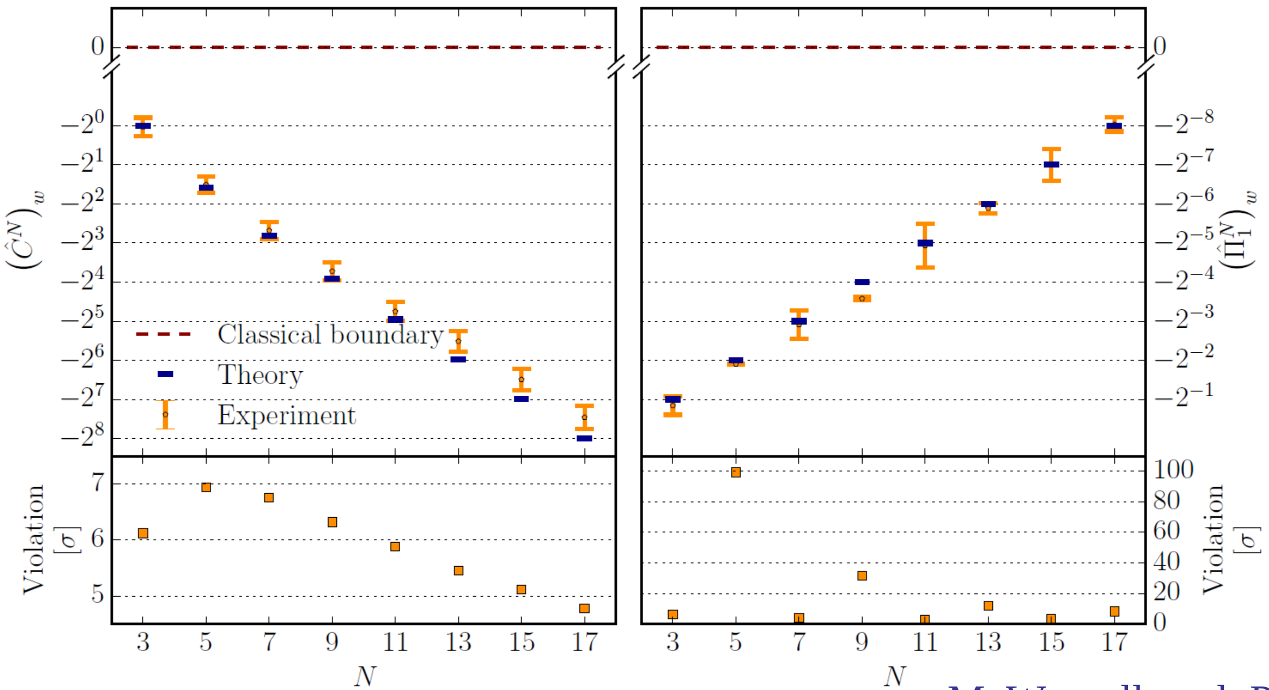
$$(\Pi_1^{(3)})_w = \prod_{n=1}^{3}\frac{1+Z_w}{2} + \prod_{n=1}^{3}\frac{1-Z_w}{2} = -\frac{1}{2} \notin [0,1]$$

Quantum contextuality in neutron interferometer



$$(\Pi_i^{(N)})_w = \prod_{n=1}^{N} \frac{1+(-1)^{x_{i,n}^{(N)}} Z_w}{2} + \prod_{n=1}^{N} \frac{1-(-1)^{x_{i,n}^{(N)}} Z_w}{2}$$

$$C^{(N)} = \mathrm{Re}\left(I - \sum_{i=1}^{2^{N-1}} s_i \, (\Pi_i^{(N)})_w\right)$$



M. Waegell et al. PRA **96**, 052131 (2017).

Weak values via weak/strong measurements

PRL **116,** 040502 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

Strong Measurements Give a Better Direct Measurement of the Quantum Wave Function

Giuseppe Vallone and Daniele Dequal
Department of Information Engineering, University of Padova, via Gradenigo 6/B, 35131 Padova, Italy
(Received 24 April 2015; revised manuscript received 24 December 2015; published 29 January 2016)

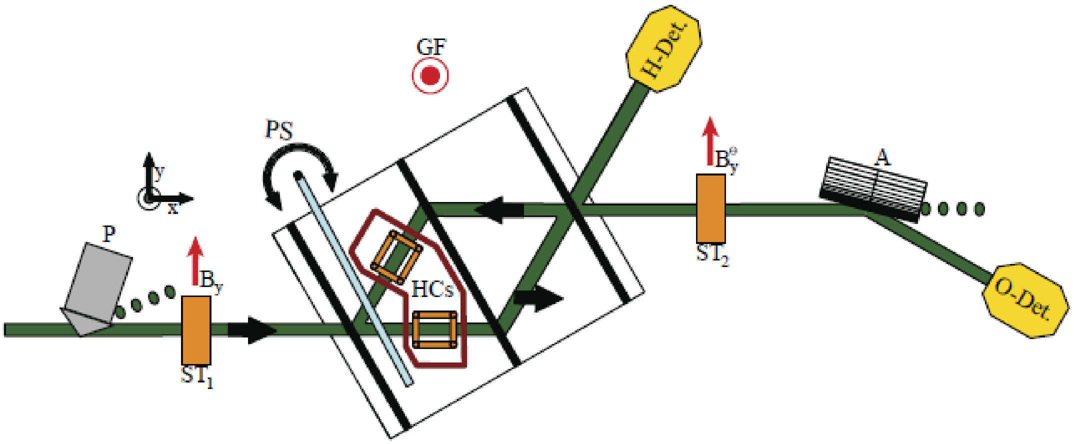
Weak measurements have thus far been considered instrumental in the so-called direct measurement of the quantum wave function [J. S. Lundeen, Nature (London) 474, 188 (2011).]. Here we show that a direct measurement of the wave function can be obtained by using measurements of arbitrary strength. In particular, in the case of strong measurements, i.e., those in which the coupling between the system and the measuring apparatus is maximum, we compared the precision and the accuracy of the two methods, by showing that strong measurements outperform weak measurements in both for arbitrary quantum states in most cases. We also give the exact expression of the difference between the original and reconstructed

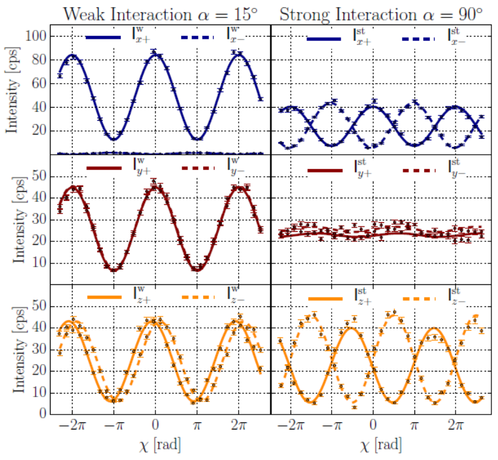
Accuracy of DWT.—In the case of DWT, the obtained wave function $\psi_{W,x}$ is an approximation of the correct wave function ψ_x. We now evaluate the accuracy of the DWT, namely, the errors arising by using Eq. (3) in place of the exact values of (4). As done in Ref. [13], we define the accuracy in terms of the trace distance $\mathcal{D}$ between the correct wave function ψ_x and the weak-value approximation $\psi_{W,x}$ [19], that for pure states reduces to $\mathcal{D} = \sqrt{1 - |\langle\psi|\psi_W\rangle|^2}$. We first give the analytical expression of $\mathcal{D}$ in terms of the original wave function and then show how $\mathcal{D}$ can be upper bounded by using the measurement outcomes.

Precision of the DWT.—An important performance parameter is the precision of the method, namely, the statistical errors on the estimated wave function. In particular, it is important to evaluate the scaling of such errors with the number of measurements. To this purpose, we evaluated the *mean square statistical error* $\delta\psi$ of the DWT and DST methods, obtained by summing the squares of the statistical error on the different ψ_x:

$$\delta\psi = \sqrt{\sum_x |\delta\psi_x|^2}. \qquad (9)$$

WV via weak/strong measurements: experiment





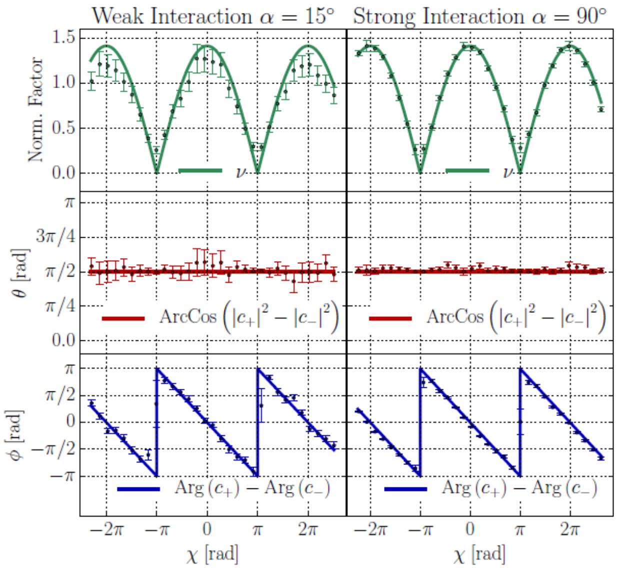


TABLE I. Quantitative comparison of precision $\bar{\sigma}$ and accuracy $\bar{\Delta}$ of the weak and the strong interaction approach.

	Precision $\bar{\sigma}$			Accuracy $\bar{\Delta}$	
	Weak	Strong		Weak	Strong
ν	0.100	0.036	ν	0.152	0.062
θ	0.191	0.065	θ	0.100	0.067
ϕ	0.355	0.159	ϕ	0.860	0.580

T. Denkmayr et al. PRL **118**, 010402 (2017).

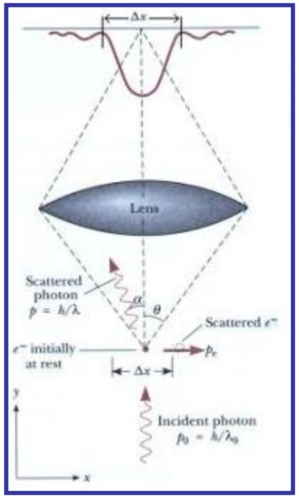
Uncertainty relation: historical

- In 1927 Heisenberg postulated an uncertainty principle:

  γ-ray thought experiment

  → $p_1 q_1 \approx h$

  with q_1 (mean error) & p_1 (discontinuous change)



- Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die unstetige Änderung von p beim Comptoneffekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

$$p_1 q_1 \sim h. \qquad (1)$$

Ozawa's Universally Valid Uncertainty Relation

PHYSICAL REVIEW A **67**, 042105 (2003)

Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement

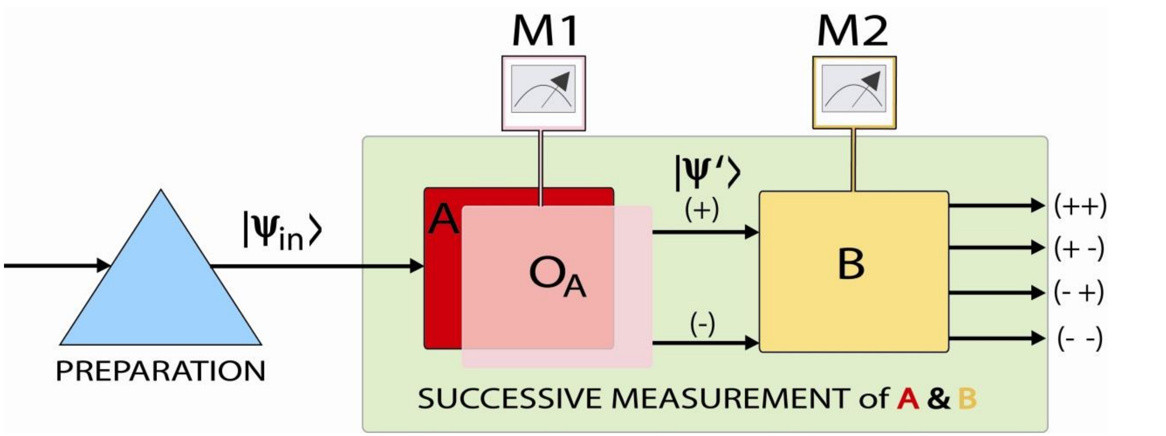
Masanao Ozawa
Graduate School of Information Sciences, Tôhoku University, Aoba-ku, Sendai, 980-8579, Japan
(Received 9 October 2002; published 11 April 2003)

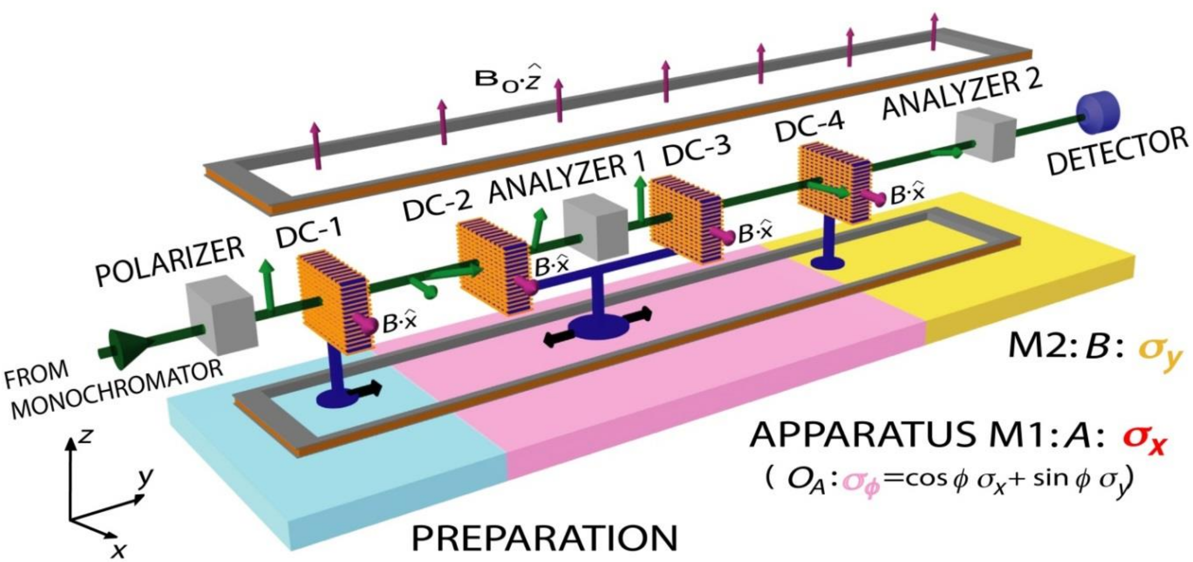
The Heisenberg uncertainty principle states that the product of the noise in a position measurement and the momentum disturbance caused by that measurement should be no less than the limit set by Planck's constant $\hbar/2$ as demonstrated by Heisenberg's thought experiment using a γ-ray microscope. Here it is shown that this common assumption is not universally true: a universally valid trade-off relation between the noise and the disturbance has an additional correlation term, which is redundant when the intervention brought by the

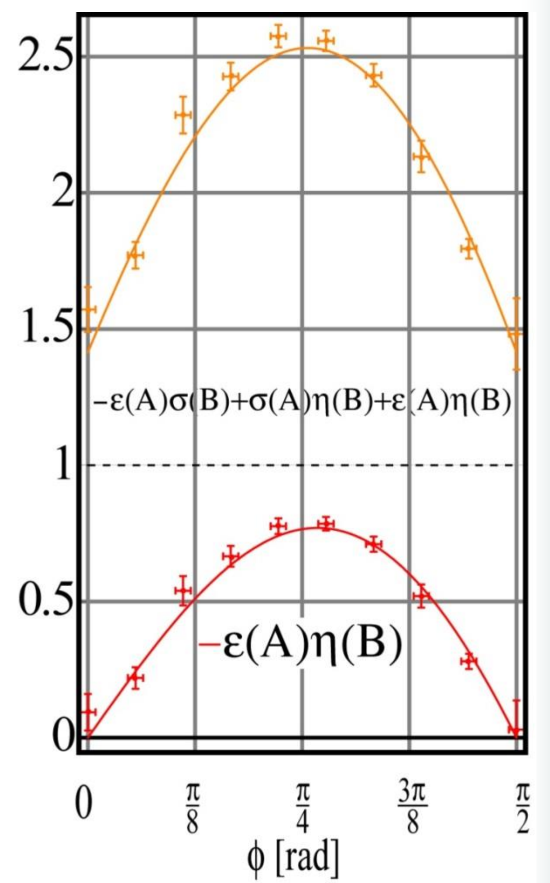
$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$$

- rigorous theoretical treatments of quantum measurements:
  - ***first term:*** error of the first measuremt, disturbance on the second measurement
  - ***second and third terms:*** crosstalks between spreads of wavefunctions and error/disturbance

Experimental test







J. Erhart et al.,
Nature Phys. **8**, 185-189 (2012)

Publications by other groups

PRL **109,** 100404 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aephraim M. Steinberg

PRL **110,** 220402 (2013) PHYSICAL REVIEW LETTERS week ending 31 MAY 2013

Experimental Test of Universal Complementarity Relations

Morgan M. Weston, Michael J. W. Hall, Matthew S. Palsson, Howard M. Wiseman, and

Centre for Quantum

(a)

(b)

ArXiv; 1304.2071

How well can one jointly measure two incompatible observables on a given quantum state?

Cyril Branciard

Centre for Engineered Quantum Systems and Scho

The University of Queensland, St Lucia,

(Dated: April 9, 2013

Heisenberg's uncertainty principle is one of the main tenets of fundamental importance for our understanding of quantum foun interpretation: although Heisenberg's first argument was that the state necessarily disturbs another incompatible observable, stand indeterminacy of the outcomes when either one or the other obse precisely Heisenberg's intuition. Even if two incompatible obser still approximate their joint measurement, at the price of intro measurement of each of them. We present a new, tight relation the error on one observable versus the error on the other. As characterize the disturbance of an observable induced by the appr derive a stronger error-disturbance relation for this scenario.

SCIENTIFIC REPORTS

OPEN

Experimental violation and reformulation of the Heisenberg's error-disturbance uncertainty relation

So-Young Baek[1]*, Fumihiro Kaneda[1], Masanao Ozawa[2] & Keiichi Edamatsu[1]

[1]Research Institute of Electrical Communication, Tohoku University, Sendai 980-8577, Japan, [2]Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan.

SUBJECT AREAS: QUANTUM MECHANICS QUANTUM METROLOGY QUANTUM INFORMATION QUANTUM OPTICS

Received 7 August 2012

Accepted 2 July 2013

Published 17 July 2013

The uncertainty principle formulated by Heisenberg in 1927 describes a trade-off between the error of a measurement of one observable and the disturbance caused on another complementary observable such that their product should be no less than the limit set by Planck's constant. However, Ozawa in 1988 showed a model of position measurement that breaks Heisenberg's relation and in 2003 revealed an alternative relation for error and disturbance to be proven universally valid. Here, we report an experimental test of Ozawa's relation for a single-photon polarization qubit, exploiting a more general class of quantum measurements than the class of projective measurements. The test is carried out by linear optical devices and realizes an indirect measurement model that breaks Heisenberg's relation throughout the range of our experimental parameter and yet validates Ozawa's relation.

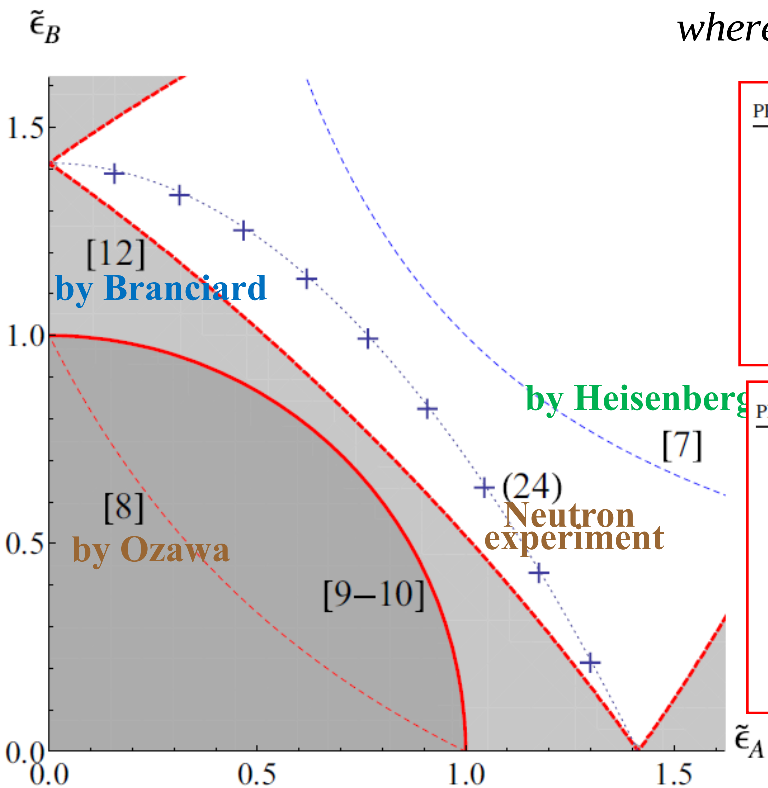
Correspondence and requests for materials

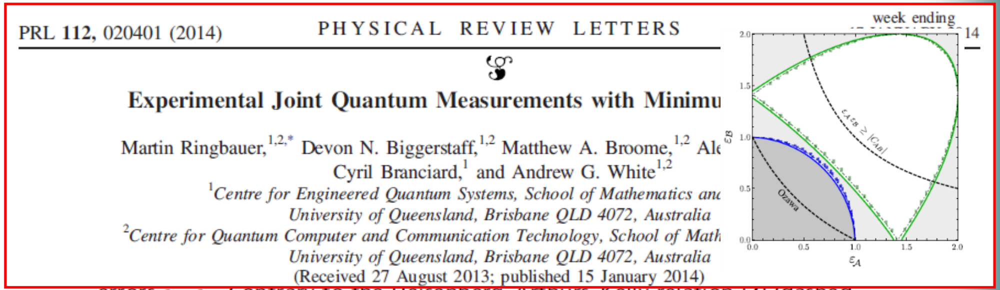
FWF

Tight relation derived by Branciard

$$\left[2\tilde{\varepsilon}(A)\tilde{\eta}(B)\sqrt{1-C^2}+\tilde{\varepsilon}(A)^2+\tilde{\eta}(B)^2\right]^{1/2}\geq C,$$

$$\textit{where } \tilde{\varepsilon}=\varepsilon\sqrt{1-\varepsilon^2/4},\ \tilde{\eta}=\eta\sqrt{1-\eta^2/4},\ C\equiv\left|\langle\psi|[A,B]|\psi\rangle\right|/2$$

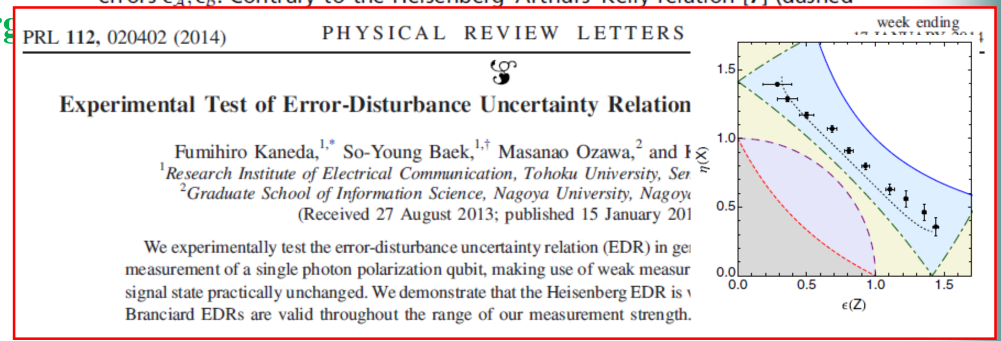



PRL 112, 020401 (2014) PHYSICAL REVIEW LETTERS week ending

Experimental Joint Quantum Measurements with Minimu

Martin Ringbauer,[1,2,*] Devon N. Biggerstaff,[1,2] Matthew A. Broome,[1,2] Ale
Cyril Branciard,[1] and Andrew G. White[1,2]
[1]Centre for Engineered Quantum Systems, School of Mathematics anc
University of Queensland, Brisbane QLD 4072, Australia
[2]Centre for Quantum Computer and Communication Technology, School of Math
University of Queensland, Brisbane QLD 4072, Australia
(Received 27 August 2013; published 15 January 2014)

errors ϵ_A, ϵ_B. Contrary to the Heisenberg–Arthurs–Kelly relation [7] (dashed


PRL 112, 020402 (2014) PHYSICAL REVIEW LETTERS week ending

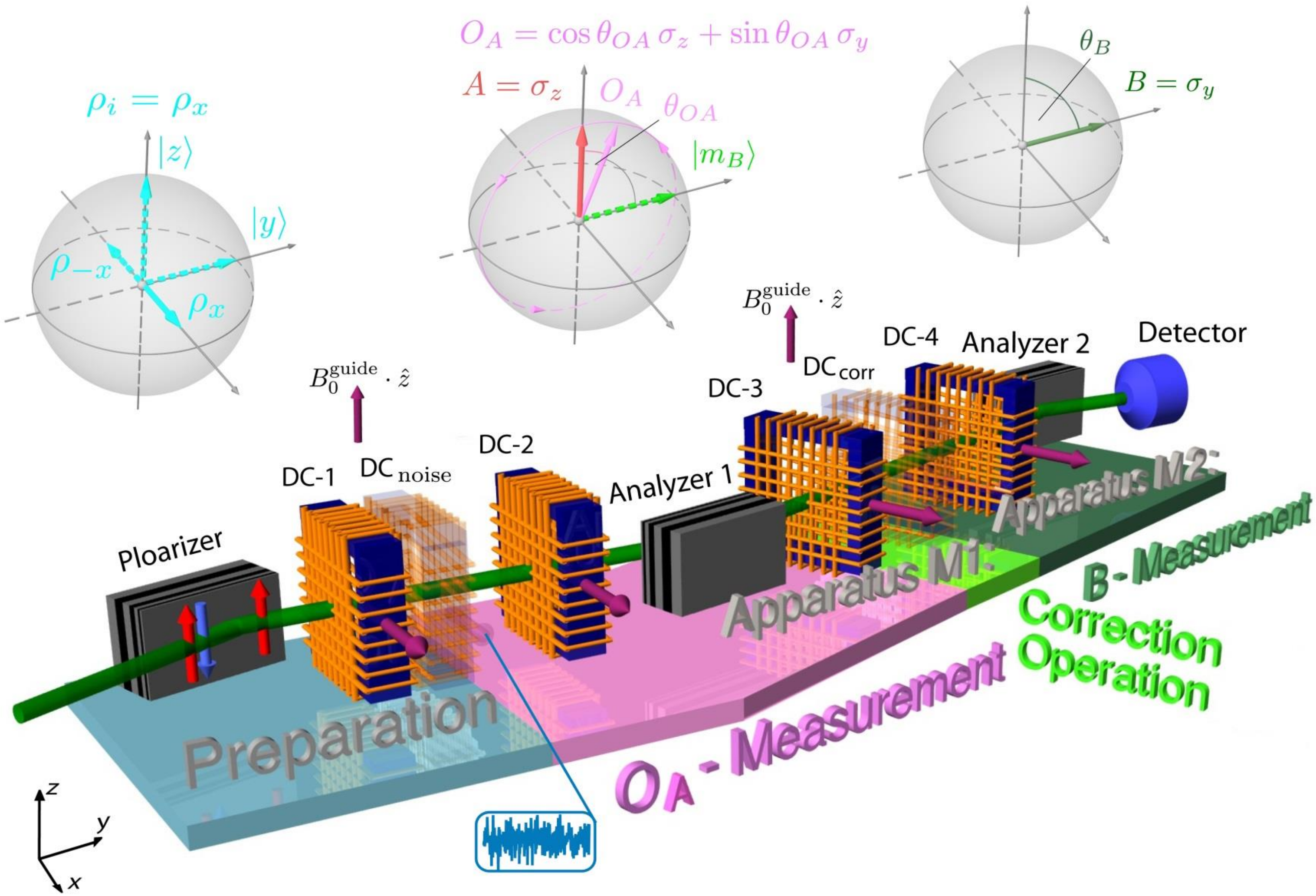
Experimental Test of Error-Disturbance Uncertainty Relation

Fumihiro Kaneda,[1,*] So-Young Baek,[1,†] Masanao Ozawa,[2] and I
[1]Research Institute of Electrical Communication, Tohoku University, Se
[2]Graduate School of Information Science, Nagoya University, Nagoy
(Received 27 August 2013; published 15 January 201

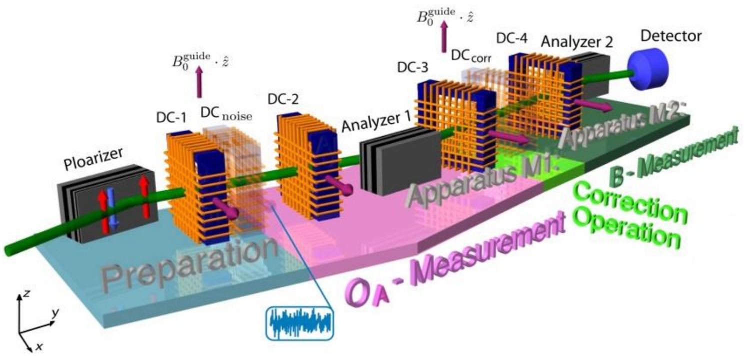
We experimentally test the error-disturbance uncertainty relation (EDR) in ge
measurement of a single photon polarization qubit, making use of weak measur
signal state practically unchanged. We demonstrate that the Heisenberg EDR is
Branciard EDRs are valid throughout the range of our measurement strength.

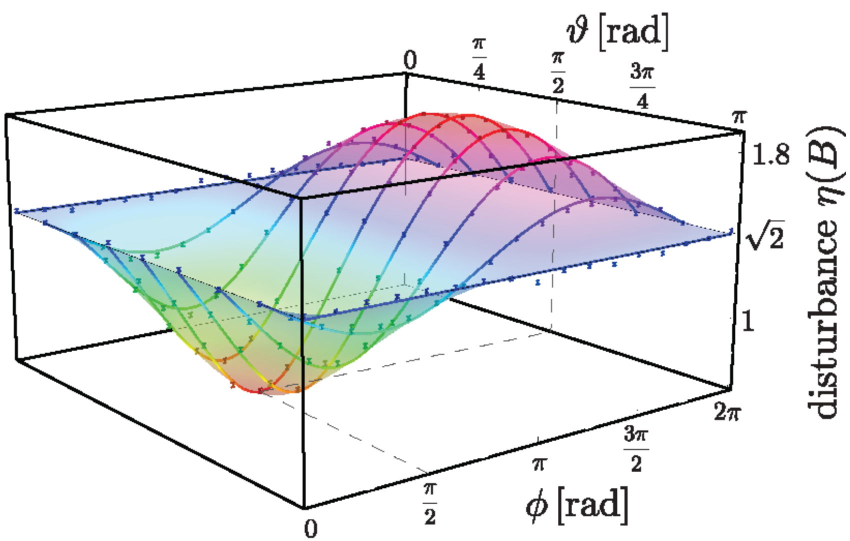
C. Branciard, Proc. Natl. Acad. Sci. U.S.A. **110**, 6742 (2013).

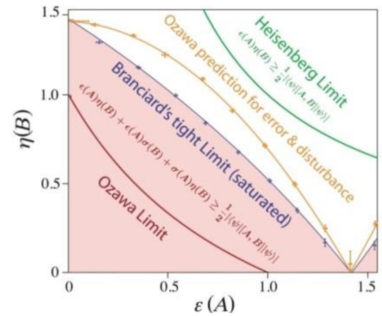
Tight relation: experimental setup

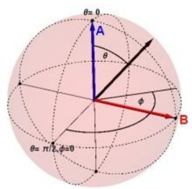


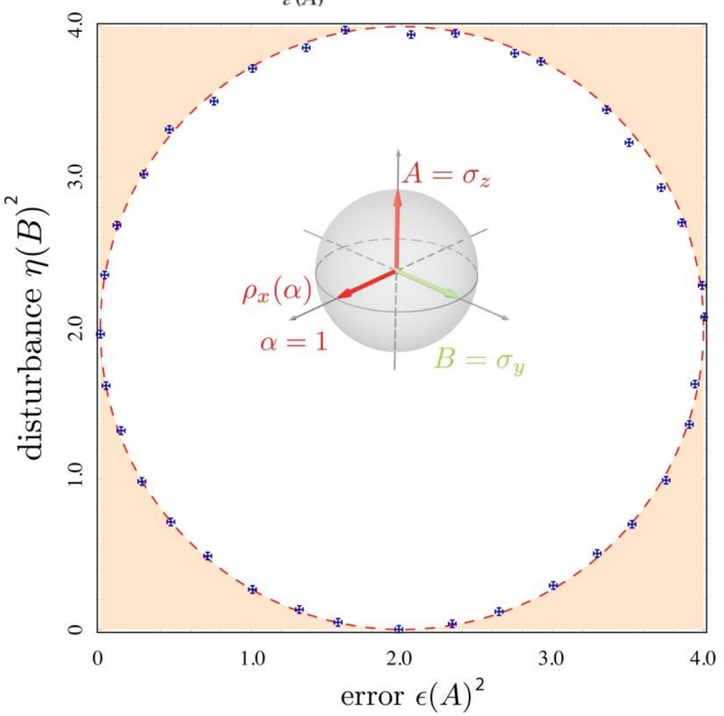
Tight relation: error-corrections



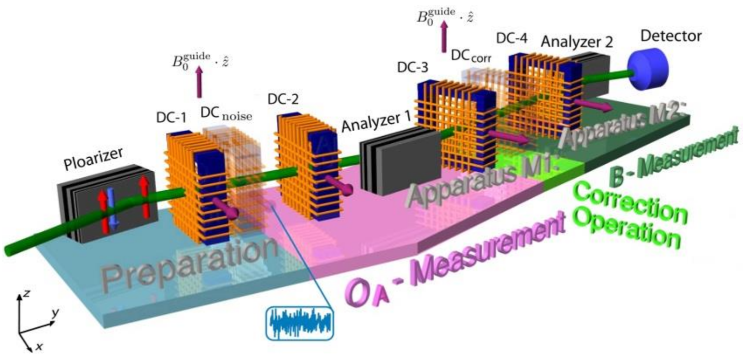


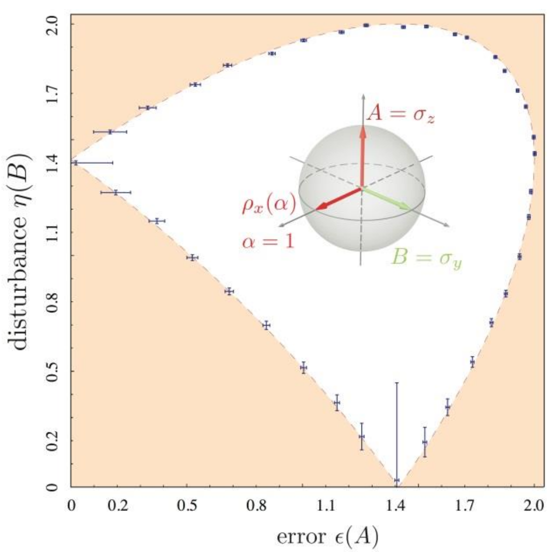


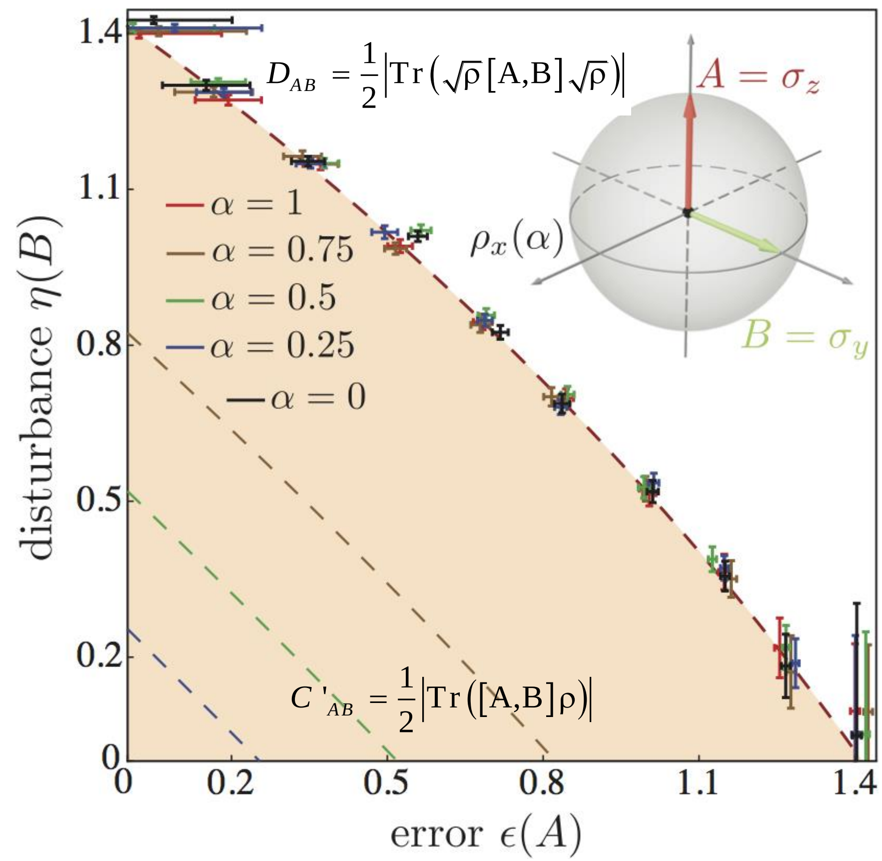




Tight relation: from a pure state to mixed states

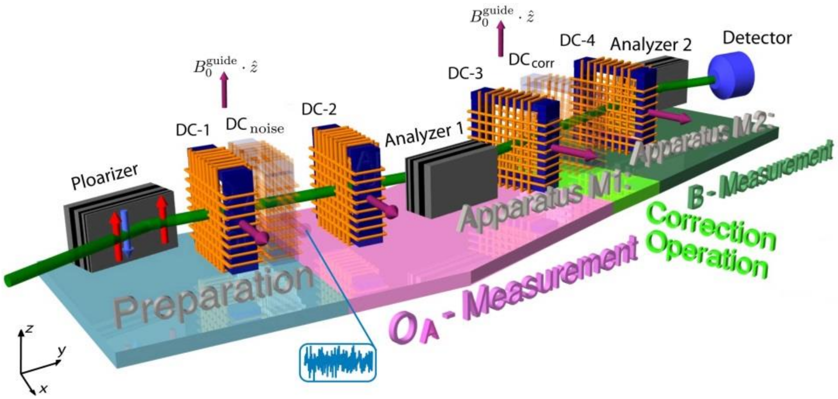


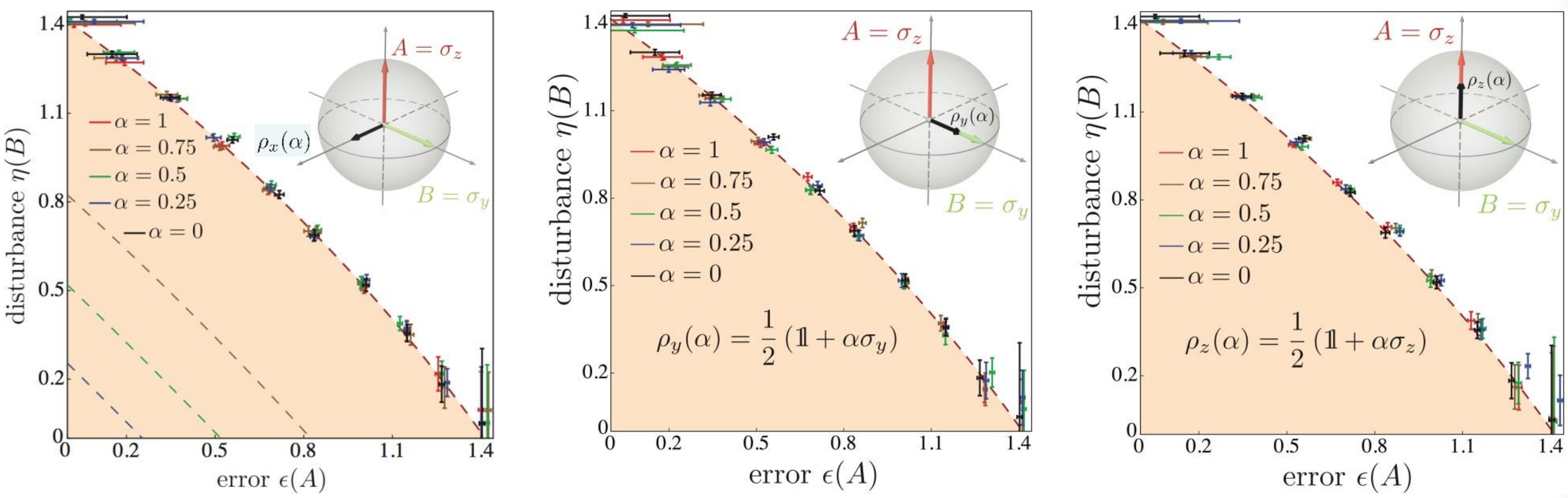




Remark: M. Ozawa, arXiv:1404.3388

Tight relation: all mixtures





B. Demirel, PRL 117, 140402 (2016).

Entropic uncertain-relation (UR)

UR for states

❖ Robertson:

$$\Delta A\,\Delta B \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$$

❖ Deutsch:

$$H(\mathcal{A}) + H(\mathfrak{B}) \geq -2\log(c)$$

$$c := \max_{j,k}|\langle a_j|b_k\rangle|$$

UR for measurements

❖ Ozawa: *arXiv:* 1404.3388v1, (2014)

$$\varepsilon(A)^2\Delta B^2 + \eta(B)^2\Delta A^2 + 2\varepsilon(A)\eta(B)\sqrt{\Delta A^2\Delta B^2 - {D_{AB}}^2} \geq {D_{AB}}^2$$

$$D_{AB} := \frac{1}{2}\mathrm{Tr}(|\sqrt{\rho}[A,B]\sqrt{\rho}|)$$

❖ Buscemi, Hall: PRL **112**, 050401 (2014)

$$N(M,A) + D(M,B) \geq -\log(c)$$

$$N(M,A) := H(\mathcal{A}|\mathcal{M})\ \&\ D(M,B) := H(\mathfrak{B}|\mathcal{M})$$

Information-theoretic Entropy

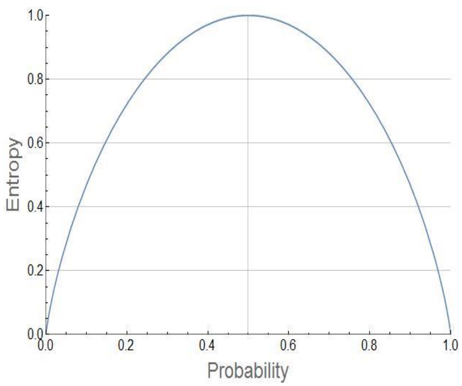
Shannon Entropy *H*:

where $A|a\rangle = \mathrm{a}|a\rangle$ for the observable A.

$$H(\mathcal{A}, |\psi\rangle) := -\sum_a p(a)\log\left(p(a)\right)$$

$$p(a) = |\langle \mathrm{a}|\psi\rangle|^2$$

Coin toss: Probability for heads or tails



p(heads) = x p(tails) = 1-x

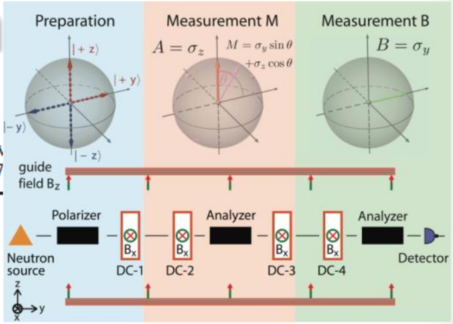
(Binary) Shannon entropy

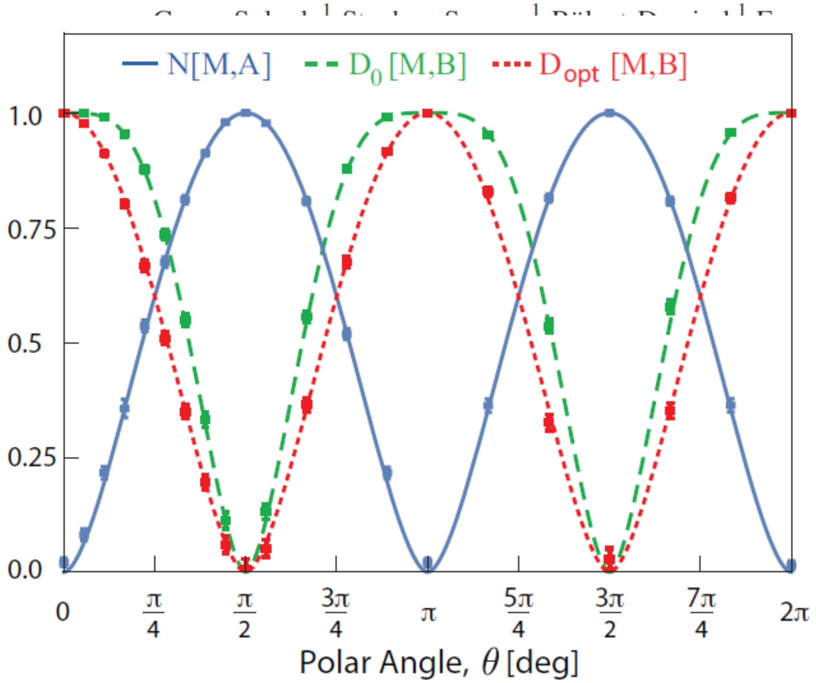
$$H(X) = -x \log(x) - (1-x) \log(1-x)$$

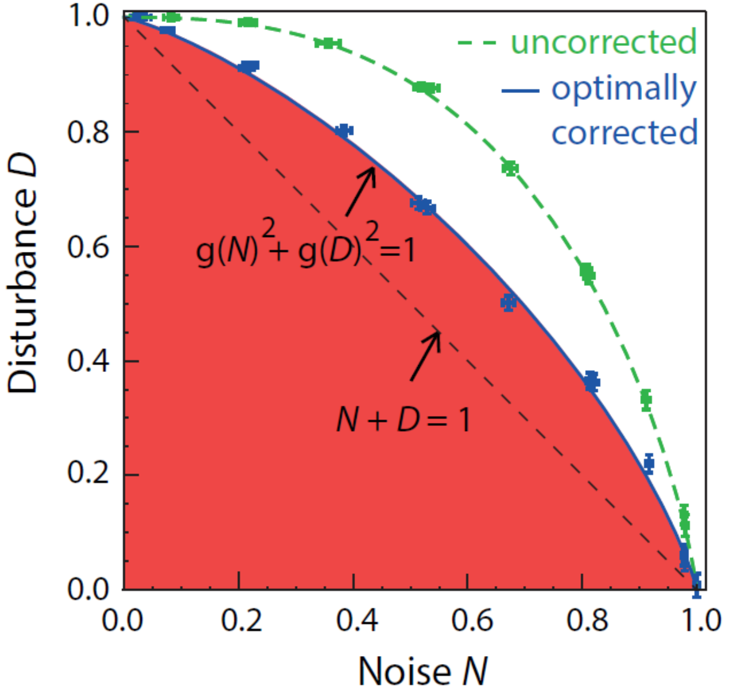
Results for entropic noise-dist. relation

PRL **115,** 030401 (2015) PHYSICAL REVIEW LETTERS

Experimental Test of Entropic Noise-Disturbance Uncertainty Relations for Spin-1/2 Measurements







Entropic noise-dist. uncertainty relation has π-periodicity !!!

Tight relation is attained.

Improvements with general POVMs

PHYSICAL REVIEW A **94**, 062110 (2016)

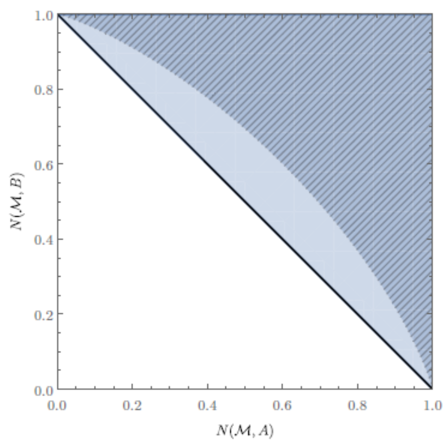
Noise and disturbance of qubit measurements: An information-theoretic characterization

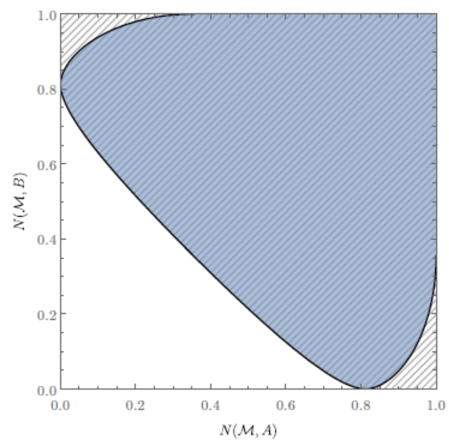
Alastair A. Abbott* and Cyril Branciard

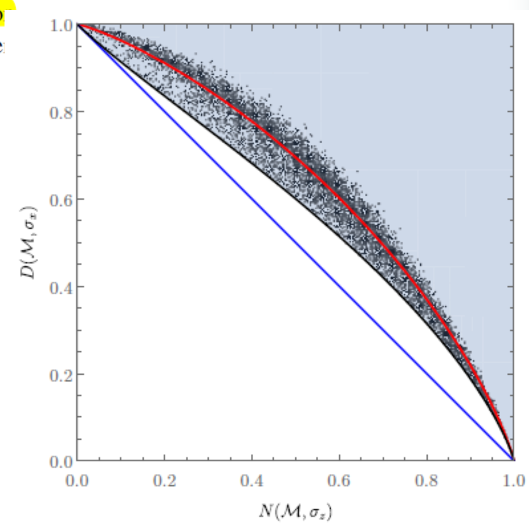
Institut Néel, CNRS and Université Grenoble Alpes, 38042 Grenoble Cedex 9, France

(Received 5 October 2016; published 12 December 2016)

Information-theoretic definitions for the noise associated with a quantum measurement and the corresponding disturbance to the state of the system have recently been introduced [F. Buscemi *et al.*, Phys. Rev. Lett. **112**, 050401 (2014)]. These definitions are invariant under relabeling of measurement outcomes, and lend themselves readily to the formulation of state-independent uncertainty relations both for the joint estimate of observables (noise-noise relations) and the noise-disturbance tradeoff. Here we derive such relations for incompatible qubit observables, which we prove to be tight in the case of joint estimates, and present progress towards fully characterizing the noise-disturbance tradeoff. In doing so, we show that the set of obtainable noise-noise values for such observables is convex, whereas the conjectured form for the set of obtainable noise-disturbance values is not. Furthermore, projective measurements are not optimal with respect to the joint-measurement noise o noise-disturbance tradeoffs. Interestingly, it seems that four-outcome measurements are needed in the forme case, whereas three-outcome measurements are optimal in the latter.





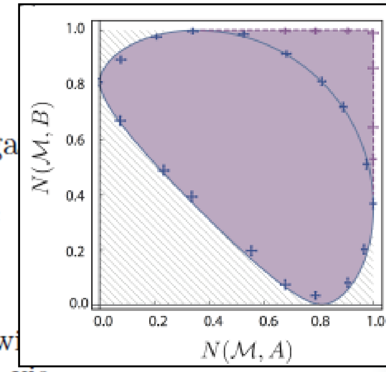


Results for entropic noise-noise relation: general POVMs

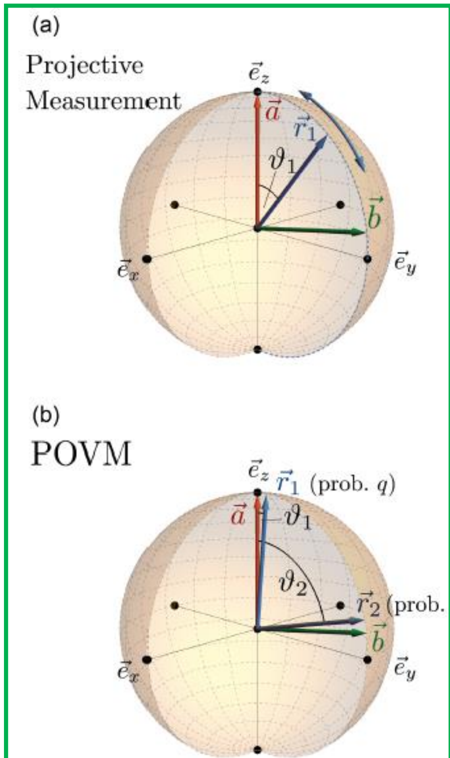
Experimental test of an entropic measurement uncertainty relation for arbitrary qubit observables

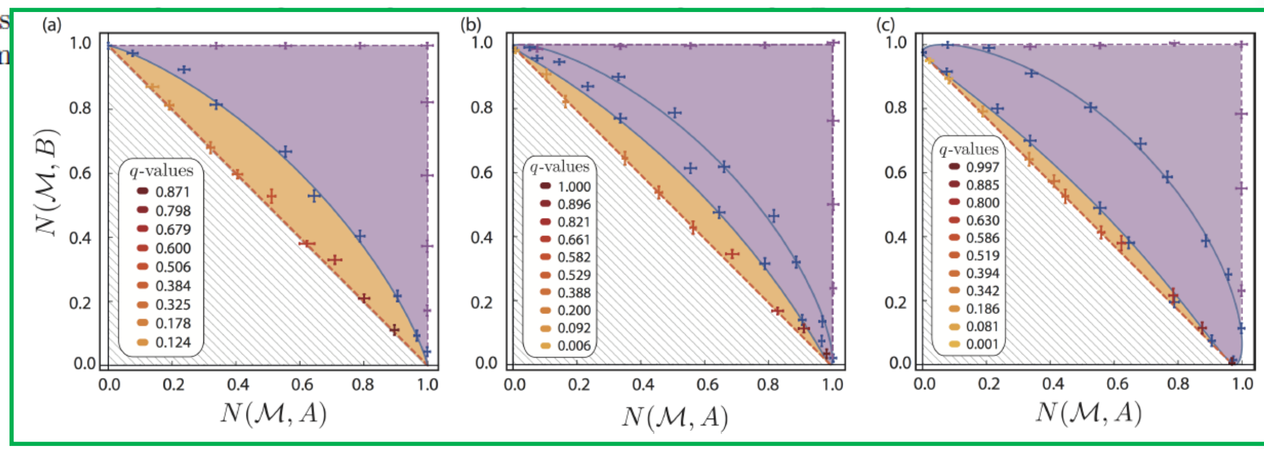
Bülent Demirel[1], Stephan Sponar[1], Alastair A. Abbott[2], Cyril Branciard[2], and Yuji Hasega
[1] *Atominstitut, TU Wien, Stadionallee 2, 1020 Vienna, Austria*
[2] *University Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France*
[3] *Department of Applied Physics, Hokkaido University, Kita-ku, Sapporo 060-8628, Japan*

(Dated: November 15, 2017)



…etic measurement uncertainty relation is experimentally tested wi… …he noise associated to the measurement of an observable is defined via …ies and a tradeoff relation between the noises for two arbitrary spin …. The optimal bound of this tradeoff is experimentally obtained for …observables. For some of these observables this lower bound can be …as… …m…





B. Demirel, arXiv:1711.05023

Concluding remarks

Neutron optical experiments are effective methods for studies of foundation of quantum mechanics.

- Quantum dyanamics:

quantum Cheshire-cat and pigeonhole effect are observed.

- Error-disturbance uncertainty relation:

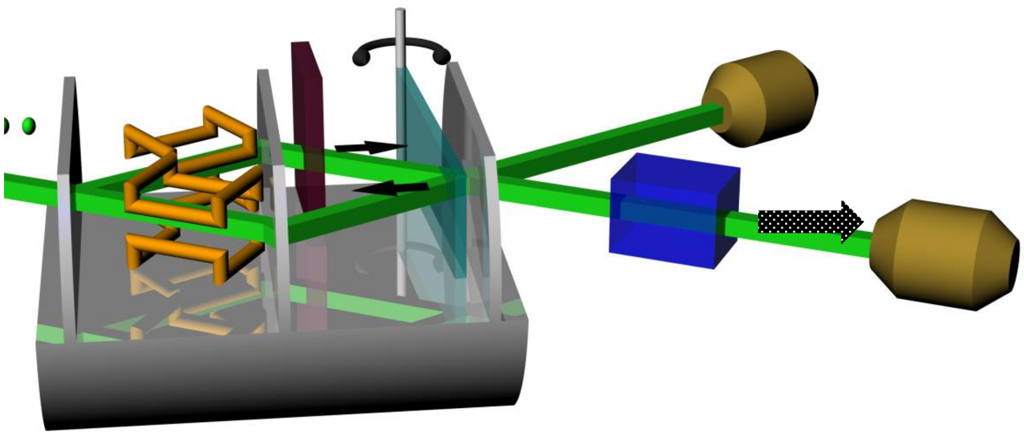
tight relations for pure/mixed states are shown.

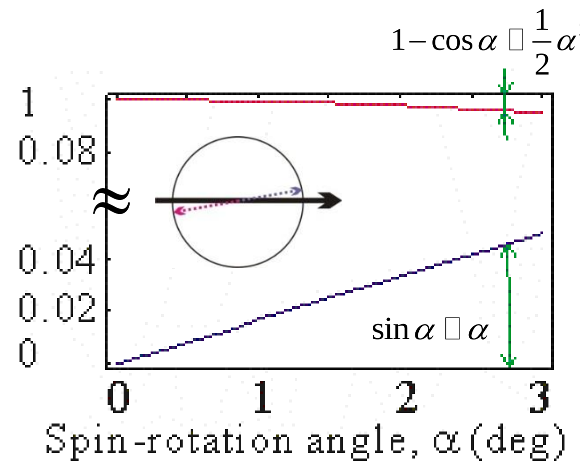
- Entropic noise-disturbance uncertainty relation:

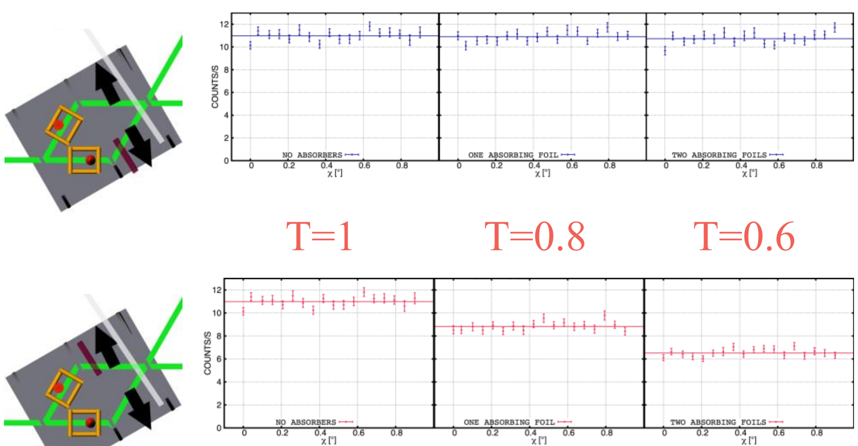
tight relations for projective/POVMs are confirmed.

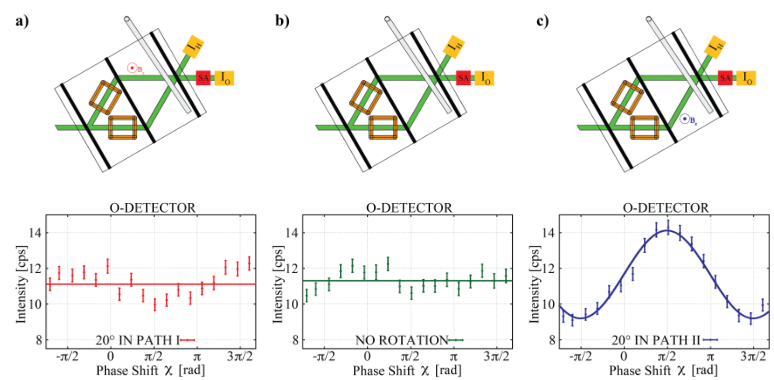
Fin !

Another view of quantum Cheshire-cat: effectiveness









Neutron interferometry

Neutrons

$m = 1.67 \times 10^{-27}$ kg

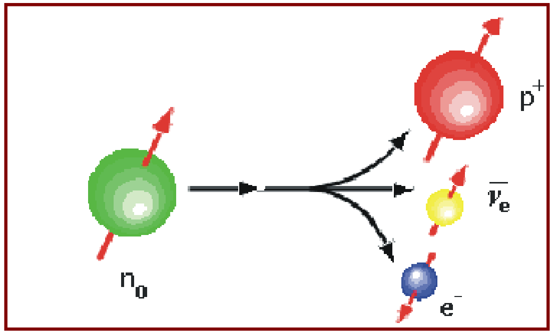
$s = \frac{1}{2}\hbar$

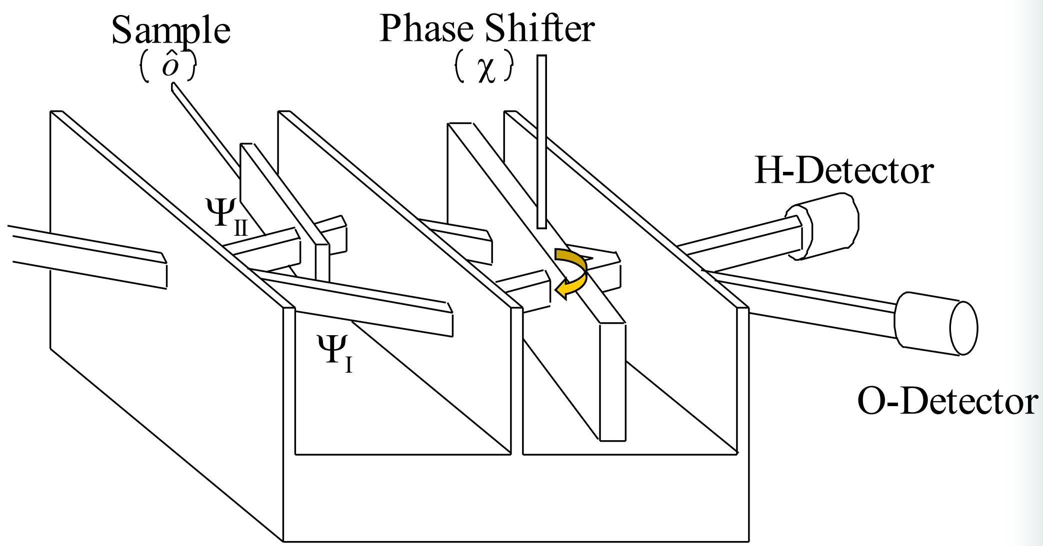
$\mu = -9.66 \times 10^{-27}$ J/T

$\tau = 887$ s

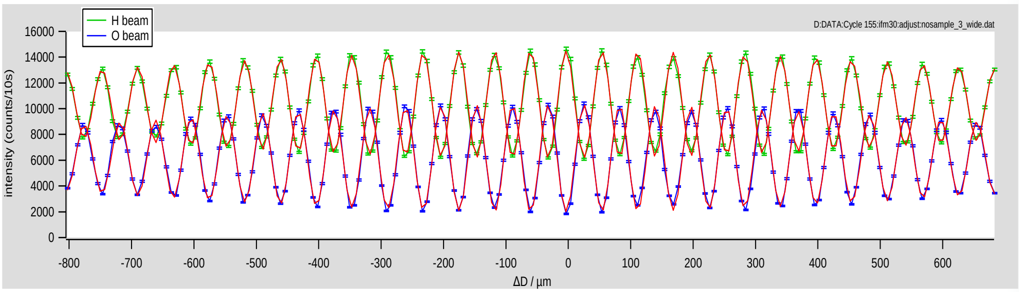
$R = 0.7$ fm

u–d–d quark structure





$$I = |\Psi_I + e^{i\chi} \cdot \hat{o} \cdot \Psi_{II}|^2$$



nature physics

LETTERS

PUBLISHED ONLINE: 15 JANUARY 2012 | DOI: 10.1038/NPHYS2194

Experimental demonstration of a universally valid error–disturbance uncertainty relation in spin measurements

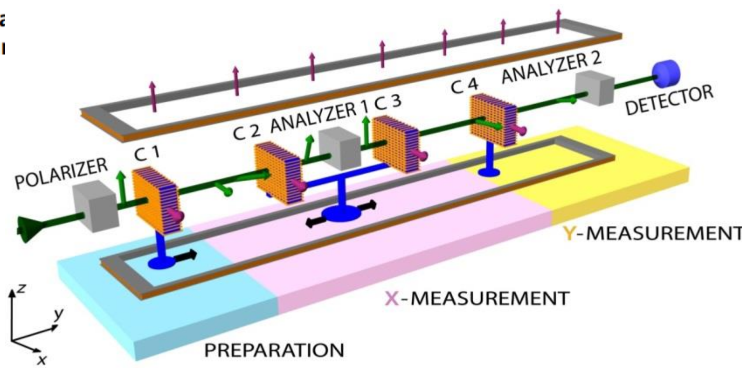
Jacqueline Erhart[1], Stephan Sponar[1], Georg Sulyok[1], Gerald Badurek[1], Masanao Ozawa[2] and Yuji Hasegawa[1]*

The uncertainty principle generally prohibits simultaneous measurements of certain pairs of observables and forms the basis of indeterminacy in quantum mechanics[1]. Heisenberg's original formulation, illustrated by the famous γ-ray microscope, sets a lower bound for the product of the measurement error and the disturbance[2]. Later, the uncertainty relation was reformulated in terms of standard deviations[3–5], where the focus was exclusively on the indeterminacy of predictions, whereas the unavoidable recoil in measuring devices has been ignored[6]. A correct formulation of the error–disturbance uncertainty rela

for a deeper u

as $\sigma(A)^2 = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2$. Note that covariance term can be added to the right-hand if squared, as discussed by Schrödinger[5]. Fc setting, this term vanishes. Robertson's relatio standard deviations has been confirmed by m ments. In a single-slit diffraction experiment[15] tion, as expressed in equation (2), has been co relation appears in squeezing coherent states and many experimental demonstrations have b

Robertson's relation (equation (2)) has a ma plications for limitation urally understood as l



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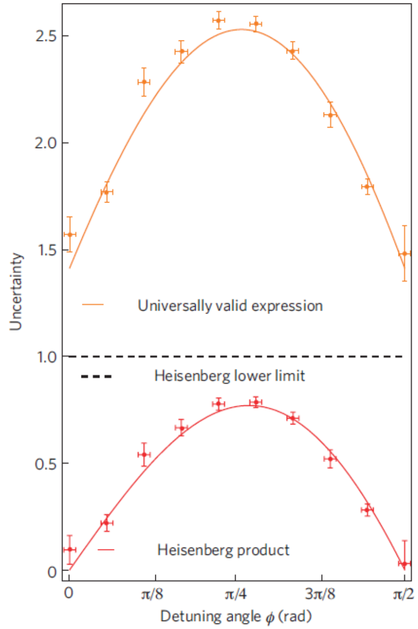

Neutron Quantum Optics generation